FINITE ELEMENT ANALYSIS OF OUT-OF-PLANE DISTORTION OF WELDED PANEL STRUCTURES

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BY

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ABSTRACT

Two dimensional distortions of a panel structure due to welding are analyzed by the finite element displacement method with an assumption of elastic deformation during welding process. Construction of stiffness matrices relevant to the welding problem is demonstrated for the one and the two dimensional cases. Computer programs are presented.

Computations have been carried out using the one dimensional experimental value of unconstrained angular change along the welded edge and its equivalent constrained welding moment as an input to the computer program.

These computed results are compared to the particular experimental values. The results indicate that an analytical approach using finite element method can predict the distortion phenomena of a complicated two dimensional structure with a reasonable degree of accuracy.

Several recommendations are made concerning further investigations aimed at evaluating relevant value of angular change and equivalent constrained welding moment.

Thesis Supervisor: Koichi Masubuchi

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NOMENCLATURE

a, b	Length of a finite element in x- and y-direction, respectively
C	Equivalent welding moment for one dimensional deflection
C_x	Equivalent welding moment along y-direction as in Figure 6
C_y	Equivalent welding moment along x-direction as in Figure 6
C	Coefficient matrix
$\underset{\sim}{D}'$	Matrix defined in equation (49)
$\underset{\sim}{D}$	Matrix defined as $\underset{\sim}{D}'/\beta$
E	Young's modulus of elasticity
$\underset{\sim}{F}_x, \underset{\sim}{F}_y, \underset{\sim}{G}, \underset{\sim}{H}, \underset{\sim}{g}$	Matrix defined in Appendix A
$\underset{\sim}{F}_x^T, \underset{\sim}{F}_y^T, \underset{\sim}{G}^T, \underset{\sim}{H}^T, \underset{\sim}{g}^T$	Transpose of the matrix $\underset{\sim}{F}_x$, $\underset{\sim}{F}_y$, $\underset{\sim}{G}$, $\underset{\sim}{H}$ and $\underset{\sim}{g}$, respectively.
I	$t^3/12$, in.^{4}/in.
$\underset{\sim}{k}$	Stiffness matrix for one rectangular finite element
$\underset{\sim}{k}_{wx}$, $\underset{\sim}{k}_{wy}$	Additional stiffness matrix due to welding defined in equation (58b) and (58c), respectively
L_x, L_y	Length of one panel structure in x- and y-directions, respectively
M, N	Number of finite element in x- and y-directions, respectively
$\underset{\sim}{P}, \underset{\sim}{P}_x, \underset{\sim}{P}_y, \underset{\sim}{P}_{\ell x}, \underset{\sim}{P}_{\ell y}$	Matrix defined in equation (64), (65), (66) and (67), respectively
$\underset{\sim}{Q}_x$, $\underset{\sim}{Q}_y$	Load matrix due to welding defined in equations (58d) and (58e), respectively
t	Thickness of plate
$\underset{\sim}{T}$	Matrix defined in Appendix A

$\underset{\sim}{T}^{-1}$	Inverse matrix of $\underset{\sim}{T}$
U_T	Total strain energy
U_p	Elastic strain energy
U_w	Welding energy
w	Deflection in plate
W	Weight of electrode consumed, gram/cm.
W_i, W_j	Weight for numerical integration
x, y, z	Ordinally coordinates for a finite element
X_i, Y_j	Numerical integration points in normalized coordinates
β	Constant defined as $Et^3/12(1-\nu^2)$
ν	Poisson's ratio
θ_o	Free joint angular change
θ	Constrained angular change
δ	Variational notation

I INTRODUCTION

A. Background

Welding is used extensively in the fabrication of many structures, including ships, airplanes, buildings, pressure vessels, etc., providing many advantages over the techniques such as riveting, casting, and forging. However, distortion problems are always encountered during welding. Distortion often prevents the achievement of design tolerances, reduces joint strength by mismatching, and imparts initial deflection into structural members.

A typical structural member is a panel structure, which is composed of a flat plate and longitudinal and transverse stiffeners fillet welded to the plate, as shown in Figure 1. Angular changes produced along the fillet welds cause out-of-plane distortion of the panel. The excessive out-of-plane distortion reduces the buckling strength of the panel that is subjected to compressive loading.(3) Corrugation failures of bottom plating in some welded cargo ships are believed to be due primarily to the reduction of buckling strength of the plating with excessive initial distortion.(15,16,17)

Welding distortion is very complicated phenomena associated with many factors:

1. Heat conductivity of weldment which controls the distribution of temperature gradient of welded metal.
2. Thermal expansion coefficient which controls the expansion and shrinkage of welded metal.

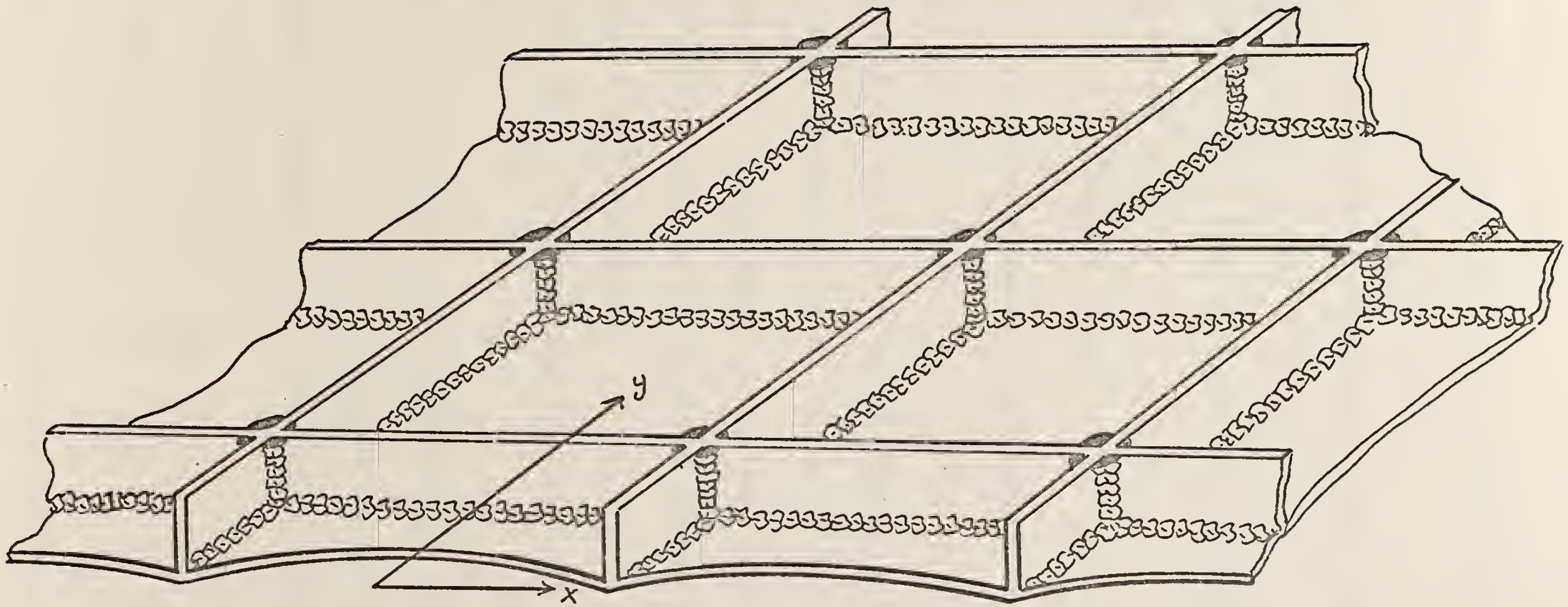


Figure 1 Panel Structure with Longitudinal and Transverse Stiffeners

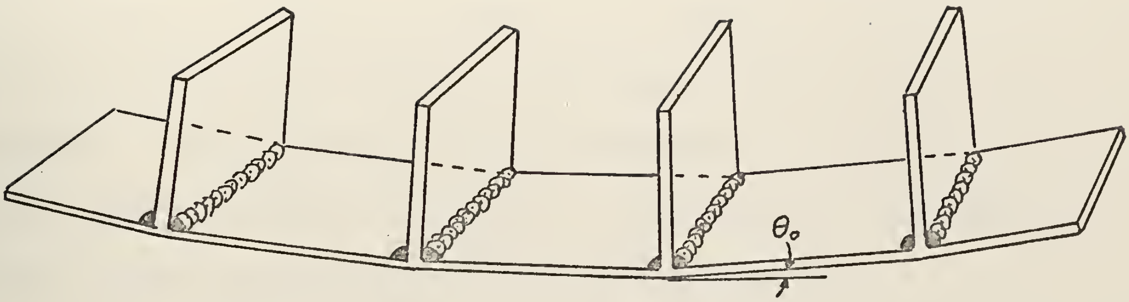
3. Moduli of elasticity which governs the rigidity against deformation.
4. Yielding strength which governs the size of the plastic zone near the welds.
5. Degree of constraint which governs the freedom of movement during weld deformation.
6. Amount of heat input used which governs the size of the melted zone.
7. Amount of weldment used.

Therefore, it is extremely difficult to analyze the welding deformation phenomena by a pure analytical way. Yet, only one dimensional deformation of panel structures has been analyzed with the assumptions of elastic deformation during welding process.[1] Obviously, welding deformations are neither pure elastic nor pure plastic phenomena, but the plastic zone near the weldments is so small compared to panel structural size that it is believed to be closer to the elastic behavior.

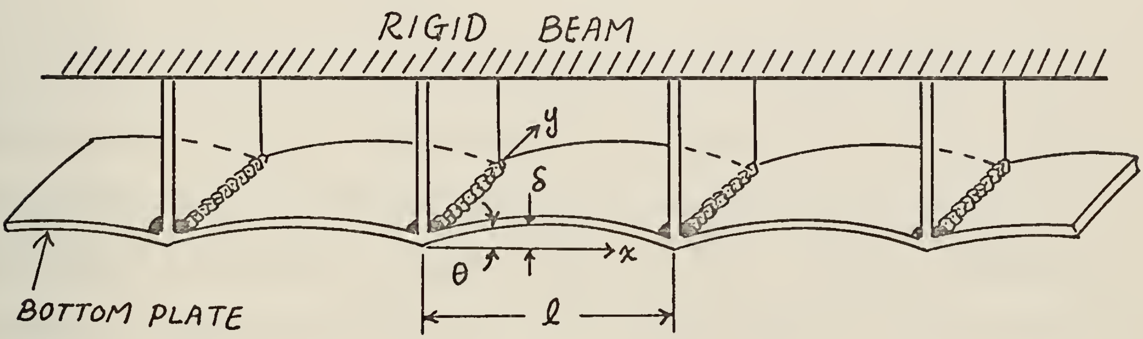
Therefore, the analytical approach of the one dimensional deformation prediction can be treated as an elsstic deformation by considering moduli of elasticity, degree of constraint, and amount of weldment used.

B. Previous Investigations

Masubuchi, et. al.[1,3] have investigated the one dimensional plate deformation during welding with a minimum strain energy concept using elastic bending theory.



a. Free Joint



b. Constrained Joint
(framed structure)

Figure 2 Distortion Caused by Angular Change in Two Types of Fillet Welded Structures

If a fillet joint is free from external constraint, the joint simply bends to a polynomical form having a knuckle at the weld as shown in Figure 2a. However, if the joint is constrained by some means, a different type of distortion is produced. For example, if the stiffeners are welded to a rigid beam, as shown in Figure 2b, the angular changes at fillet welds cause wavy or arc-form distortion.

In the simplest case in which sizes of all welds are the same, the distortions of all spans are equal and the distortion, w, which can be expressed as follows:

$$\delta/\ell = [1/4 - (x/\ell - 1/2)^2] \cdot \theta \qquad (1)$$

where δ = deflection of plate

θ_o = angular change at a fillet weld with constraint

ℓ = length of span

The amount of angular change, θ, in constrained structures is smaller than that in a free joint, θ_o. This indicates that a certain amount of energy is necessary to decrease the angular change from θ_o to θ. If the necessary energy is represented by U_w, it may be given by the following equation.

$$U_w = \int_0^{\theta_o-\theta} \frac{dU_w}{d(\theta_o-\theta)} \, d(\theta_o-\theta) \qquad (2)$$

On the other hand, the strain energy stored in the constrained plate, U_p, can be expressed using the elastic beam theory.

$$U_p = \frac{E'I}{\ell} \theta^2 \qquad (3)$$

where $E' = E/1 - \nu^2$

$I = t^3/12$

Since U_p increases but on the contrary U_w decreases as the constrained angle θ increases, the condition for equilibrium of this system requires that the total strain energy $U_t = U_w + U_p$ should be minimum. Furthermore, for the simplicity of the problem, the ratio of incremental welding energy change to angular change is assumed to be linear as in the following equation.

$$\frac{dU_w}{d(\theta_o-\theta)} = C(\theta_o-\theta) \tag{4}$$

where C = a coefficient determined by the weight of the deposited metal and by the welding procedures.

From equations (2) and (4), welding energy per unit length of width can be expressed in terms of C, θ, and θ_o,

$$U_w = \frac{C}{2}(\theta_o-\theta)^2 \tag{5}$$

Accordingly, the condition of equilibrium is as follows:

$$\frac{\partial U_w}{\partial\theta} = C(\theta_o-\theta) + 2\frac{E'I}{\ell} = 0 \tag{6}$$

From equation (6), relation between θ and θ_o can be expressed,

$$\theta = \frac{\theta_o}{1 + \frac{2E'I}{\ell}/C} \tag{7}$$

The values of C can be determined by using relationship obtained by experimentally measuring θ_o and θ. Empirical formula for the calculation of C has been proposed by

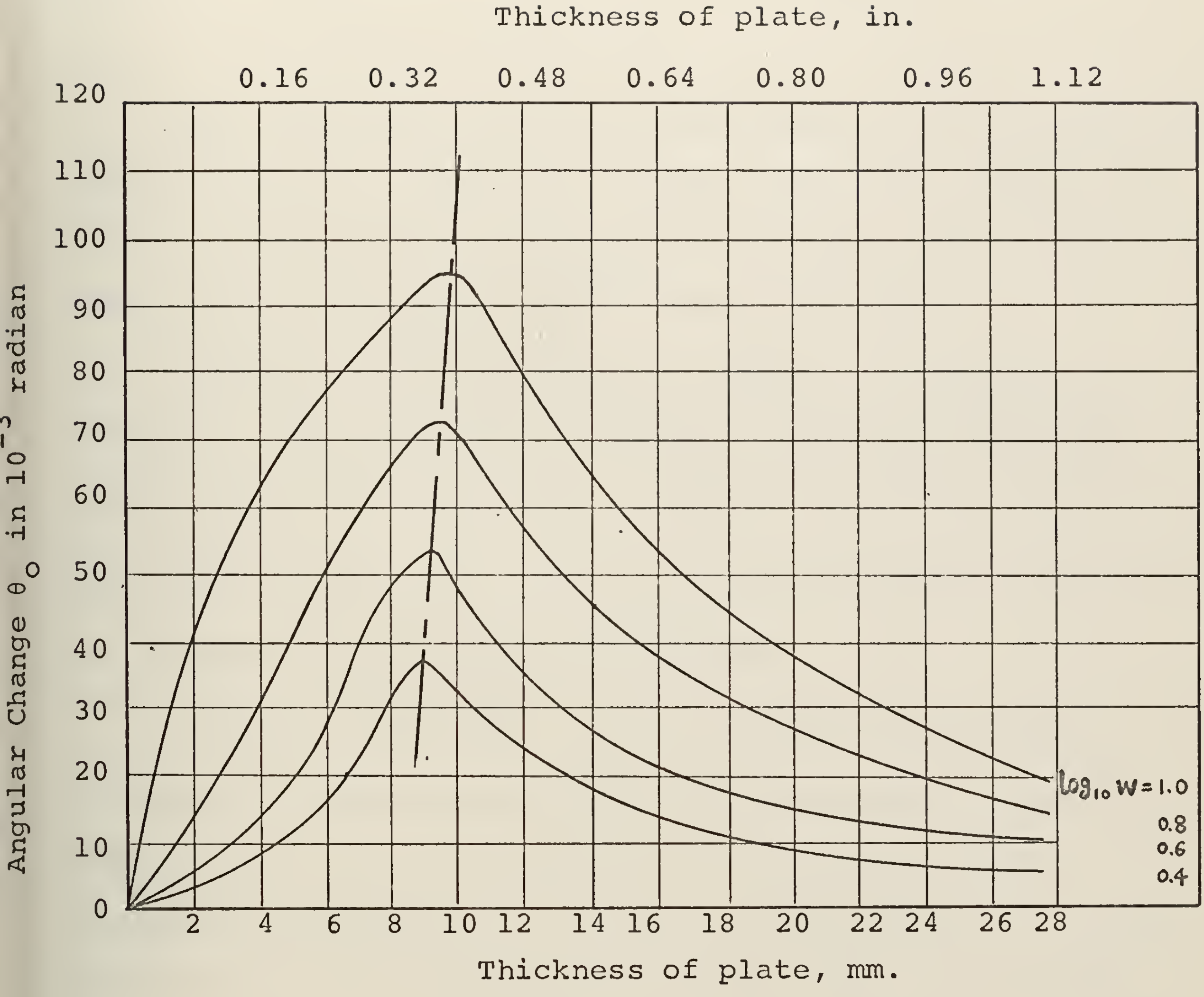


Figure 3 Variation of Angular Change of Free Fillet Welds, θ_o, As a Function of Plate Thickness, t, and Weight of Electrode Consumed per Weld Length, W.

Masubuchi.[3]

$$C = t^4/(1+W/5) \qquad \text{kg-mm/mm radian} \qquad (8)$$

where W = weight of electrode consumed, gram/cm.

t = thickness of plate, mm.

The experimental results of determining the unconstrained angular change as a function of plate thickness, t(mm), and weight of electrode consumed per weld length, W(gr/cm), have been given by Hirai, et. al.[2] as in Figure 3.

Therefore, for a given plate geometry and welding con condition, the deflection of the one dimensional case can easily be estimated by using equations (1), (7), (8), and Figure 3.

C. Aim and Purpose of Present Studies

As stated previously, welding deformation has been analyzed only for the one dimensional case, but in reality most of the structural members are encountered in two dimensional deformation, in which the deformation, w, varies along the x- and y-directions as well as the unconstrained angular change, θ, and equivalent constrained welding moment, C. With the idea of previous investigations which state that the total strain energy in the plate during welding deformation becomes minimum, it is possible to extend the one dimensional deformation analysis to the two dimensional case. However, the simple mathematical solution using plate bending equation is rather difficult due to the uncertainty of the

equivalent load and the boundary conditions relevant to the welding deformation problem.

The difficulties can be easily overcome by formulating the problem into a finite element model because it can be self-adjusted to take care of the equivalent load and equivalent sitffness matrix due to welding. Furthermore, in this way the relation of deformation phenomena to the constrained angular change, θ, and the equivalent constrained welding moment C can be easily seen.

Therefore, it is intended to handle the two dimensional deformation with the finite element displacement technique.

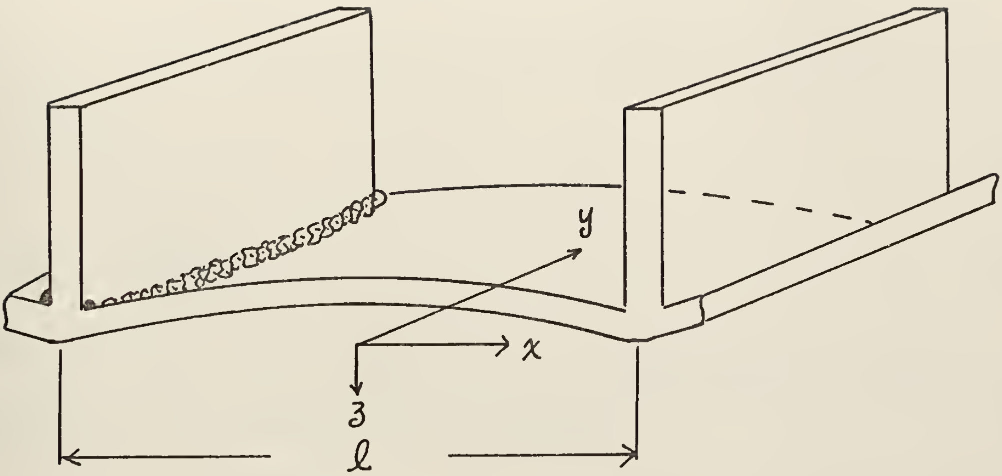
II FORMULATION OF FINITE ELEMENT EQUATION

A. One Dimensional Case

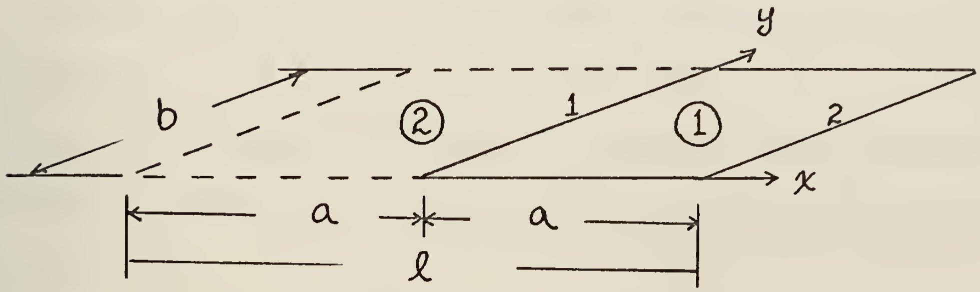
In this study, computer programs are programmed only for the two dimensional finite element analysis, but the one dimensional finite element formulations are presented to visualize the inside of the complex matrix assembling and computational procedure; furthermore, to compare the results to that of the previous investigators.

For the one dimensional case, as shown in Figure 2b, if deflections are elastic and small, the deflection can be calculated using elastic theory when the welding equivalent loads and the boundary conditions are prescribed. Even though the deformations are assumed to be elastic for the welded structural case, the loading and boundary conditions to be applied for this problem are not clear.

As introduced by Masubuchi, et. al.[1], if the concept of equivalent constrained welding moment, C(lb-in/in rad.) is being used, the uncertainty of loading conditions may be overcome; for the boundary condition it may be safe to assume to be simply supported for which only the angular change along the welded edge can be allowed. Furthermore, for simplicity of the problem, the deflections are considered only for the case of one span length of a simply supported beam to eliminate the complexity of interaction due to the statically indeterminate structural effects, as in Figure 4a.



a. Simplified one-span beam element



b. Local coordinate and nodal points for a finite beam element

Figure 4 One-span Beam Element and the Finite Element Local Coordinate

For the one dimensional plane stress, the strain energy can be expressed simply by:

$$U_p = 1/2 \iiint \varepsilon_x \sigma_x dxdydz \tag{9}$$

Using the stress-strain and strain-curvature relation for the beam element,

$$\sigma_x = E\varepsilon_x \tag{10a}$$

$$\varepsilon_x = -z \frac{d^2w}{dx^2} \tag{10b}$$

Therefore, the strain energy can be expressed in terms of deflection and rigidity of the plate:

$$U_p = \frac{EI}{2} \int_0^a \left(\frac{d^2w}{dx^2}\right)^2 dx \tag{11}$$

where

$$I = 2 \int_0^{t/2} \int_0^b z^2 dzdy = \frac{bt^3}{12}$$

On the other hand, the welding energy can be expressed in terms of θ_o, θ, and C, as in equation (5). For this problem it is more convinient to express the welding energy in terms of deflection rather than the angular changes. From equation (5),

$$U_w = \int_0^a \int_0^b \frac{c}{2}(\theta_o - \theta)^2 \, dydx$$

the constrained angular change can be expressed by the equation

$$\theta = \partial w/\partial x \tag{12}$$

Therefore, the welding energy expressed,

$$U_w = \frac{bc}{2} \int_0^a (\theta_o - \frac{\partial w}{\partial x})^2 \, dx \tag{13}$$

where the values of C and θ_o are constant which will be determined by the particular welding condition and the geometry.

The equilibrium condition for the system requires that the total strain energy has to be minimized, in other words, the variation of the total strain energy has to be zero, which becomes

$$\delta U_T = \delta U_p + \delta U_w = 0 \tag{14}$$

using equations (11) and (13), equation (14) becomes

$$\delta U_T = \delta \left[\frac{EI}{2} \int_0^a \left(\frac{d^2 w}{dx^2} \right)^2 dx \right] + \delta \left[\frac{bc}{2} \int_0^a (\theta_o - \frac{dw}{dx})^2 \, dx \right] = 0$$

But the variation of the constant term is zero, therefore,

$$EI \int_0^a (\frac{d^2 \delta w}{dx^2}) (\frac{d^2 w}{dx^2}) dx + \frac{bc}{2} \int_0^a (\frac{d\delta w}{dx}) (\frac{dw}{dx}) dx - bc \int_0^a (\frac{d\delta w}{dx}) dx \tag{15}$$

For a beam element as shown in Figure 4b, the deflection can be assumed to be a general cubic function of x,

$$w = c_1 + c_2 x + c_3 x^2 + c_4 x^3 \tag{16}$$

and when the nodal displacements and the nodal angles are the unknown variables which are to be calcualted, the coefficients (c_1, c_2, c_3, c_4) in equation (16) can exclusively be expressed in terms of the nodal displacements and angles considering the requirements of continuity at the nodal points

$$w(x=0) = w_1 = c_1$$

$$\frac{\partial w}{\partial x}(x=0) = w_{x_1} = c_2$$

$$w(x=a) = w_2 = c_1 + c_2 a + c_3 a^2 + c_4 a^3$$

$$\frac{\partial w}{\partial x}(x=a) = w_{x_2} = c_2 + 2c_3 a + 3c_4 a^2$$

The above requirements can be expressed in the matrix form,

$$\underset{\sim}{q} = \underset{\sim}{T}\ \underset{\sim}{C} \tag{17}$$

where

$$\underset{\sim}{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & a & a^2 & a^3 \\ 0 & 1 & 2a & 3a^2 \end{bmatrix} \tag{17a}$$

$$\underset{\sim}{C} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \quad (17b) \qquad \text{and} \qquad \underset{\sim}{q} = \begin{bmatrix} w_1 \\ w_{x_1} \\ w_2 \\ w_{x_2} \end{bmatrix} \quad (17c)$$

From equation (17), $\underset{\sim}{C}$ can be expressed

$$\underset{\sim}{C} = \underset{\sim}{T}^{-1}\ \underset{\sim}{q} \tag{18}$$

If the new matrix $\underset{\sim}{G}$ is defined such that

$$\underset{\sim}{G} = \underset{\sim}{T}^{-1} \tag{19}$$

then,

$$\underset{\sim}{C} = \underset{\sim}{G}\ \underset{\sim}{q} \tag{20}$$

where

$$\underset{\sim}{G} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3/a^2 & -2/a & 3/a^2 & -1/a \\ 2/a^3 & 1/a^2 & -2/a^3 & 1/a^2 \end{bmatrix} \tag{21}$$

The second derivative of deflection function becomes,

$$\frac{d^2w}{dx^2} = 2c_3 + 6c_4x$$

which is in matrix form

$$\frac{d^2w}{dx^2} = \underset{\sim}{H}\ \underset{\sim}{C} \tag{22}$$

where $\underset{\sim}{H}$ is the matrix defined by

$$\underset{\sim}{H} = (0 \quad 0 \quad 2 \quad 6x) \tag{23}$$

then,

$$\left(\frac{d^2\delta w}{dx^2}\right)\left(\frac{d^2w}{dx^2}\right) = \delta(\underset{\sim}{H}\ \underset{\sim}{C})^T(\underset{\sim}{H}\ \underset{\sim}{C}) = (\delta\underset{\sim}{C}^T)(\underset{\sim}{H}^T\underset{\sim}{H}\ \underset{\sim}{C})$$

but, using the equation (20),

$$\left(\frac{d^2\delta w}{dx^2}\right)\left(\frac{d^2w}{dx^2}\right) = (\delta\underset{\sim}{q}^T)(\underset{\sim}{G}^T\ \underset{\sim}{H}^T\ \underset{\sim}{H}\ \underset{\sim}{G}\ \underset{\sim}{q}) \tag{24}$$

The matrix expression for the first derivative of the deflection w can be expressed in the same manner,

$$\frac{dw}{dx} = c_2 + 2c_3x + 3c_4x^2$$

using $\underset{\sim}{F}$ and $\underset{\sim}{C}$

$$\frac{dw}{dx} = \underset{\sim}{F}\ \underset{\sim}{C} \tag{25}$$

where

$$\underset{\sim}{F} = (0 \quad 1 \quad 2x \quad 3x^2) \tag{26}$$

Finally, using equation (20)

$$\frac{d\delta w}{dx} = (\delta \underset{\sim}{q}^T)(\underset{\sim}{G}^T \; \underset{\sim}{F}^T) \tag{27}$$

and

$$\left(\frac{d\delta w}{dx}\right)\left(\frac{dw}{dx}\right) = (\delta \underset{\sim}{q}^T)(\underset{\sim}{G}^T \; \underset{\sim}{F}^T \; \underset{\sim}{F} \; \underset{\sim}{G} \; \underset{\sim}{q}) \tag{28}$$

From equations (24), (27) and (28), the minimum energy equation (15) can be written in the matrix form,

$$\begin{aligned} &EI \int_0^a (\delta \underset{\sim}{q}^T)(\underset{\sim}{G}^T \; \underset{\sim}{H}^T \; \underset{\sim}{H} \; \underset{\sim}{G} \; \underset{\sim}{q})dx \\ &+bc \int_0^a (\delta \underset{\sim}{q}^T)(\underset{\sim}{G}^T \; \underset{\sim}{F}^T \; \underset{\sim}{F} \; \underset{\sim}{G} \; \underset{\sim}{q})dx \\ &-bc \int_0^a (\delta \underset{\sim}{q}^T)(\underset{\sim}{G}^T \; \underset{\sim}{F}^T)dx = 0 \end{aligned} \tag{29}$$

Noticing that the matrix $\underset{\sim}{q}$, $\underset{\sim}{q}^T$, $\underset{\sim}{G}$, and $\underset{\sim}{G}$ is not the function of x, but only of the function of dimension of the plate, and displacement and angle at the nodal points; therefore, equation (29) can be simplified to

$$\begin{aligned} &\delta \underset{\sim}{q}^T \{EI \; \underset{\sim}{G}^T \; [\int_0^a (\underset{\sim}{H}^T \; \underset{\sim}{H})dx]\underset{\sim}{G} \; \underset{\sim}{q} \\ &+bc \; \underset{\sim}{G}^T \; [\int_0^a (\underset{\sim}{F}^T \; \underset{\sim}{F})dx]\underset{\sim}{G} \; \underset{\sim}{q} \\ &-bc \; \underset{\sim}{G}^T \; [\int_0^a (\underset{\sim}{F}^T)dx]\} = 0 \end{aligned} \tag{30}$$

in which the only integrations necessary to be carried out are the bracketed quantities, which become after integration,

$$\int_0^a (\underset{\sim}{H}^T \underset{\sim}{H})\,dx = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4a & 6a^2 \\ 0 & 0 & 6a^2 & 12a^3 \end{bmatrix} \tag{31}$$

$$\int_0^a (\underset{\sim}{F}^T \underset{\sim}{F})\,dx = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2a & 3a^2 \\ 0 & 2a & 4a^2 & 6a^3 \\ 0 & 3a^2 & 6a^3 & 9a^4 \end{bmatrix} \tag{32}$$

$$\int_0^a (\underset{\sim}{F}^T)\,dx = \begin{bmatrix} 0 \\ 1 \\ 2a \\ 3a^2 \end{bmatrix} \tag{33}$$

Equation (30) is the final equation for one element formulated by the finite element displacement method. The first quantity is defined as a stiffness matrix for an element,[11] the second is the additional stiffness matrix due to welding, and the third is the equivalent load due to welding. They are as follows:

Stiffness matrix for an element,

$$\underset{\sim}{k} = EI\ \underset{\sim}{G}^T [\int_0^a (\underset{\sim}{H}^T\ \underset{\sim}{H})dx]\underset{\sim}{G}$$

$$= EI \begin{pmatrix} 12/a^3 & 6/a^2 & -12/a^3 & 6/a^2 \\ 6/a^2 & 4/a & -6/a^2 & 2/a \\ -12/a^3 & -6/a^2 & 12/a^3 & -6/a^2 \\ 6/a^2 & 2/a & -6/a^2 & 4/a \end{pmatrix} \qquad (34)$$

Additional stiffness matrix due to welding,

$$\underset{\sim}{k}_w = bc\ \underset{\sim}{G}^T [\int_0^a (\underset{\sim}{F}^T\ \underset{\sim}{F})dx]\underset{\sim}{G}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & bc \end{pmatrix} \qquad (35)$$

Load matrix due to welding,

$$Q = bc\ \underset{\sim}{G}^T [\int_0^a \underset{\sim}{F}^T\ dx]$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \\ bc\theta_o \end{pmatrix} \qquad (36)$$

At this stage, assembling of equation (30) has to be considered when the one-span beam is divided by many finite elements, but the one-span beam element of Figure 4a is chosen to be two finite elements as in Figure 4b; then, only element 1 can be used to predict the deflection and angular

change at the nodal points by symmetric argument. In this case, the simultaneous equations to be solved are (from equation (30)),

$$\delta w_1 [\frac{6EI}{a^2} (\frac{2}{a} w_1 + w_{x_1} - \frac{2}{a} w_2 + w_{x_2})] = 0$$

$$\delta w_{x_1} [\frac{2EI}{a} (\frac{3}{a} w_1 + 2w_{x_1} - \frac{3}{a} w_2 + w_{x_2})] = 0$$

$$\delta w_2 [\frac{6EI}{a^2} (- \frac{2}{a} w_1 - w_{x_1} + \frac{2}{a} w_2 - w_{x_2})] = 0$$

$$\delta w_{x_2} [\frac{2EI}{a} (\frac{3}{a} w_1 + w_{x_1} - \frac{3}{a} w_2 + (2 + \frac{abc}{2EI}) w_{x_2} - bc\theta_o)] = 0$$

(37a-d)

From the simply supported boundary condition at node 2, $w_2 = 0$; therefore, $\delta w_2 = 0$.

From symmetry condition at node 1, $w_{x_1} = 0$; therefore, $\delta w_{x_1} = 0$.

Furthermore, the displacement at node 1 and the angle at node 2 is not known, which means that the variations of these variables are not zero; therefore, the only way to satisfy equation (37a) and (37d) is the bracketed value in each equation has to be zero, which leads to final equation for w and w_{x_2},

$$\frac{2}{a} w_1 + w_{x_2} = 0$$

$$\frac{3}{a} w_1 + (2 + \frac{abc}{2EI}) w_{x_2} = 0 \qquad (38a\text{-}b)$$

From equation (38a-b), displacement at the middle point and angular change at the edge expressed as,

$$w_1 = \frac{-a\theta_o}{2(1 + EI/abc)} \tag{39}$$

$$w_{x_2} = \frac{\theta_o}{2(1 + EI/abc)} \tag{40}$$

It is worth to compare the results of equations (6) and (7) with equations (39) and (40). In equations (39) and (40),

$$\frac{EI}{abc} = \frac{bt^3/12}{abc} = \frac{t^3/12}{ac}$$

and $a = \ell/2$ is to be used. Then,

$$w_1 = \frac{-\ell\theta_o}{4(1 + 2EI/\ell c)} \tag{41}$$

$$w_{x_2} = \frac{\theta_o}{1 + 2EI/\ell c} \tag{42}$$

where I is defined by unit width basis,[1,6] which is $I = t^3/12$.

Equation (42) agrees with equation (7), as also does equation (41), if $x = \ell/2$ and θ of equation (7) is plugged into equation (1). The negative sign of equation (41) means that the coordinate and the positive angle at the edge has to be chosen, as in Figure 4a.

As shown, the results from the finite element method is exactly the same expression proposed by Masubuchi, et.al.[1,3] for the one dimensional deformation, and in this way, the equivalent loads and the stiffness matrix due to welding become easily visualized.

B. Two Dimensional Case

1. Formulation of the Problem

In the two dimensional analysis, all procedures are the same as that of the one dimensional analysis except that the integrations are carried out by the numerical methods and the methods of assembling are mentioned. The deflection may be approximated by the bi-cubic function for one element,[12,14] as in Figure 5.

$$\begin{aligned} w = {} & c_1 + c_2 x + c_3 y + c_4 xy + c_5 x^2 + c_6 y^2 + c_7 x^3 \\ & + c_8 x^2 y + c_9 xy^2 + c_{10} y^3 + c_{11} x^3 y + c_{12} x^2 y^2 \\ & + c_{13} xy^3 + c_{14} x^3 y^2 + c_{15} x^2 y^3 + c_{16} x^3 y^3 \end{aligned} \tag{43a}$$

$$\begin{aligned} \frac{\partial w}{\partial x} = {} & c_2 + c_4 y + 2c_5 x + 3c_7 x^2 + 2c_8 xy + c_9 y^2 \\ & + 3c_{11} x^2 y + 2c_{12} xy^2 + c_{13} y^3 + 3c_{14} x^2 y^2 \\ & + 2c_{15} xy^3 + 3c_{16} x^2 y^3 \end{aligned} \tag{43b}$$

$$\begin{aligned} \frac{\partial w}{\partial y} = {} & c_3 + c_4 x + 2c_6 y + c_8 x^2 + 2c_9 xy + 3c_{10} y^2 \\ & + c_{11} x^3 + 2c_{12} x^2 y + 3c_{12} xy^2 + 2c_{14} x^3 y \\ & + 3c_{15} x^2 y^2 + 3c_{16} x^3 y^2 \end{aligned} \tag{43c}$$

$$\begin{aligned} \frac{\partial^2 w}{\partial x \partial y} = {} & c_4 + 2c_8 x + 2c_9 y + 3c_{11} x^2 + 4c_{12} xy \\ & + 3c_{13} y^2 + 6c_{14} x^2 y + 6c_{15} xy^2 + 9c_{16} x^2 y^2 \end{aligned} \tag{43d}$$

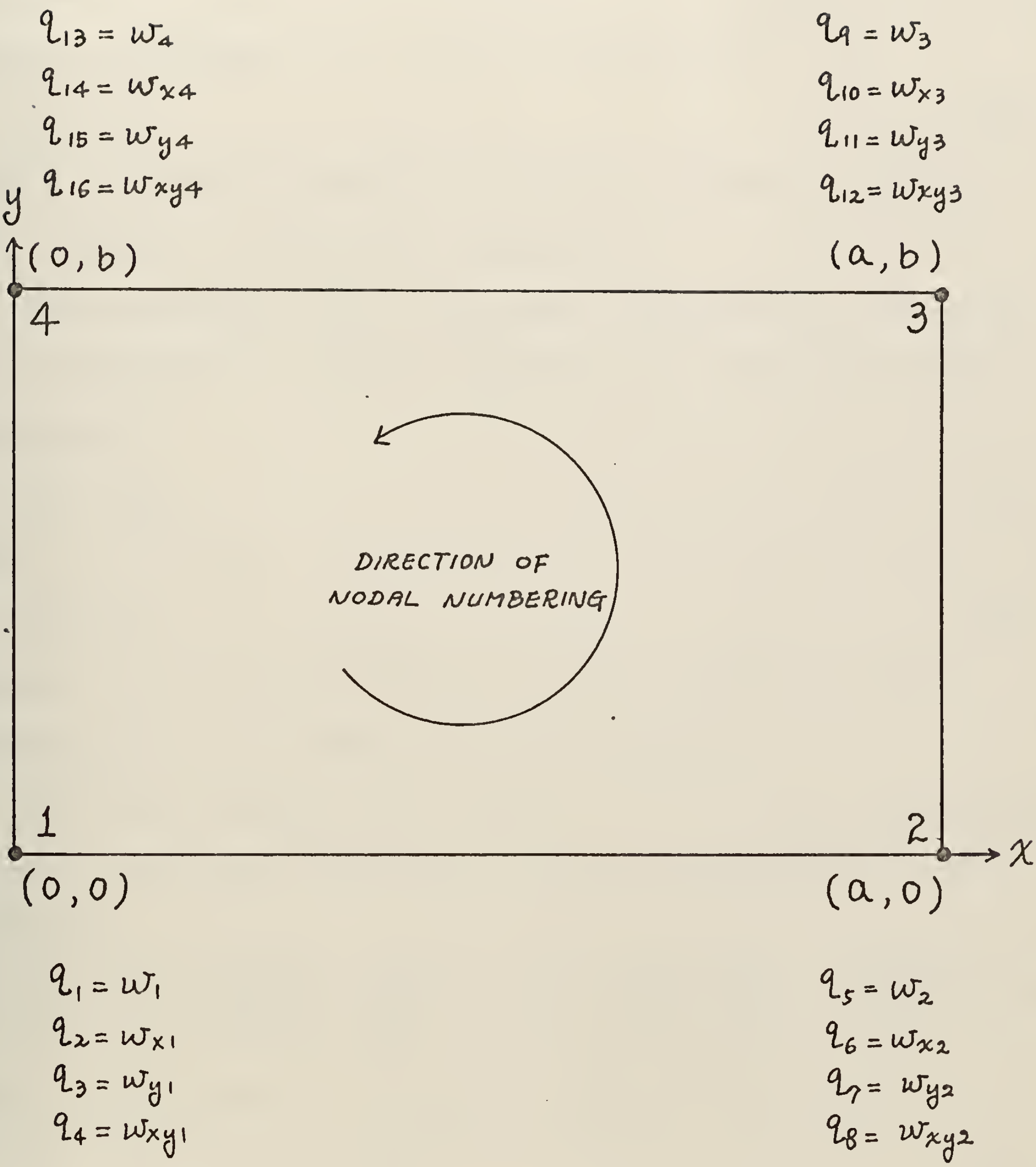


Figure 5 Local Coordinate of a Two dimensional Rectangular Finite Element and Generalized Displacement Numbering

From equation (43a-d), the matrix equation can be defined such that at each nodal point, the generalized displacements (deflections, angles in x and y, and twisting angles) are expressed in terms of the geometry of the element and the some unknown coefficients C_i, which is the form of,

$$\underset{\sim}{q} = \underset{\sim}{T}\,\underset{\sim}{C} \tag{44}$$

where $\underset{\sim}{T}$ is 16 by 16 matrix, $\underset{\sim}{C}$ is 16 by 1 matrix, and $\underset{\sim}{q}$ is 16 by 1 matrix defined as in Appendix A.

From equation (44), the unknown coefficient matrix, $\underset{\sim}{C}$, can be expressed in terms of geometry and the generalized displacements,

$$\underset{\sim}{C} = \underset{\sim}{G}\,\underset{\sim}{q} \tag{45}$$

where

$$\underset{\sim}{G} = \underset{\sim}{T}^{-1} \tag{46}$$

However, the actual calculation of the inverse of $\underset{\sim}{T}$ is carried out by the computer in this analysis.

From the elastic plate theory,[8,9,10] the strain energy stored in the plate during the elastic deformation is expressed by

$$U_p = \frac{D}{2}\int_0^a\int_0^b\left\{\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right]^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x\partial y}\right]^2\right\} dxdy$$

This can be written in the matrix form,[7] which is

$$U_p = \frac{1}{2}\int_0^a\int_0^b \underset{\sim}{K}^T\,\underset{\sim}{D}\,\underset{\sim}{K}\; dydx \tag{47}$$

where

$$\underset{\sim}{K} = -\begin{bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\[2ex] \dfrac{\partial^2 w}{\partial y^2} \\[2ex] 2\dfrac{\partial^2 w}{\partial x \partial y} \end{bmatrix} = -\underset{\sim}{H}\ \underset{\sim}{C} \tag{48}$$

in which $\underset{\sim}{H}$ is given by Appendix A,

$$\underset{\sim}{D}' = \frac{Et^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = \beta \cdot \underset{\sim}{D} \tag{49}$$

in which

$$\beta = \frac{Et^3}{12(1-\nu^2)}$$

and

$$\underset{\sim}{D} = \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Therefore, equation (48) becomes after taking the variation with respect to generalized displacement,

$$\delta U_p = \int_0^a \int_0^b (\delta \underset{\sim}{K}^T)\, \beta\, \underset{\sim}{D}\, \underset{\sim}{K}\, dydx$$

$$= \int_0^a \int_0^b (\delta \underset{\sim}{C}^T)(\underset{\sim}{H}^T\ \beta\ \underset{\sim}{D}\ \underset{\sim}{H}\ \underset{\sim}{C})\, dydx$$

$$= \delta \underset{\sim}{q}^T\ \beta\ \underset{\sim}{G}^T \left[\int_0^a \int_0^b \underset{\sim}{H}^T\ \underset{\sim}{D}\ \underset{\sim}{H}\ dydx\right] \underset{\sim}{G}\ \underset{\sim}{q} \tag{50}$$

The welding energy for the elements shown in Figure 6b can be written as,

$$U_w = \int_0^a \frac{C_y}{2} (\theta_o - \theta_y)^2 dx + \int_0^b \frac{C_x}{2} (\theta_o - \theta_x)^2 dy$$

Therefore,

$$U_w = \frac{C_y}{2} \int_0^a (\theta_o - \frac{\partial w}{\partial y})^2 dx + \frac{C_x}{2} \int_0^b (\theta_o - \frac{\partial w}{\partial x})^2 dy \tag{51}$$

which becomes,

$$U_w = \left[\frac{C_y}{2} \int_0^a (\frac{\partial w}{\partial y})^2 dx + \frac{C_x}{2} \int_0^b (\frac{\partial w}{\partial x})^2 dy\right]$$

$$- \left[C_y \theta_o \int_0^a \frac{\partial w}{\partial y} dx + C_x \theta_o \int_0^a \frac{\partial w}{\partial x} dy\right]$$

$$+ \left[\frac{1}{2}(C_y + C_x)\theta_o^2\right] \tag{52}$$

and taking the variation,

$$\delta U_w = \left[C_y \int_0^a \left(\frac{\partial \delta w}{\partial y}\right)\left(\frac{\partial w}{\partial y}\right) dx + C_x \int_0^b \left(\frac{\partial \delta w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right) dy\right]$$

$$- \left[C_y \theta_o \int_0^a \left(\frac{\partial \delta w}{\partial y}\right) dx + C_x \theta_o \int_0^a \left(\frac{\partial \delta w}{\partial x}\right) dy\right] \tag{53}$$

in which the first bracketed term is the additional stiffness matrix due to welding, and the second is equivalent load matrix for the welding. In matrix form, equation (53) can be written, if some matrices defined as below:

$$\frac{\partial w}{\partial x} = \underset{\sim}{F}_y \underset{\sim}{C} = \underset{\sim}{C}^T \underset{\sim}{F}_y^T \tag{54a}$$

$$\frac{\partial w}{\partial y} = \underset{\sim}{F}_x \underset{\sim}{C} = \underset{\sim}{C}^T \underset{\sim}{F}_x^T \tag{54b}$$

where the subscripts x and y denote that the integration should be carried out along the x- and y-direction, respectively, and the matrix $\underset{\sim}{F}_x$ and $\underset{\sim}{F}_y$ are defined in Appendix A. Therefore,

$$\left(\frac{\partial\,\delta w}{\partial y}\right)\left(\frac{\partial w}{\partial y}\right) = (\delta \underset{\sim}{C}^T)(\underset{\sim}{F}_y^T \underset{\sim}{F}_y \underset{\sim}{C}) = (\delta \underset{\sim}{q}^T)\underset{\sim}{G}^T \underset{\sim}{F}_y^T \underset{\sim}{F}_y \underset{\sim}{G} \underset{\sim}{q} \tag{55a}$$

$$\left(\frac{\partial\,\delta w}{\partial x}\right)\left(\frac{\partial w}{\partial x}\right) = (\delta \underset{\sim}{C}^T)(\underset{\sim}{F}_x^T \underset{\sim}{F}_x \underset{\sim}{C}) = (\delta \underset{\sim}{q}^T)\underset{\sim}{G}^T \underset{\sim}{F}_x^T \underset{\sim}{F}_x \underset{\sim}{G} \underset{\sim}{q} \tag{55b}$$

Using equations (54a-b) and (55a-b), the welding term can be written as:

$$\begin{aligned}
\delta U_w = {} & \delta \underset{\sim}{q}^T\, c_y\, \underset{\sim}{G}^T \left[\int_0^a (\underset{\sim}{F}_x^T \underset{\sim}{F}_x)\,dx\right] \underset{\sim}{G}\, \underset{\sim}{q} \\
& + \delta \underset{\sim}{q}^T\, c_x\, \underset{\sim}{G}^T \left[\int_0^b (\underset{\sim}{F}_y^T \underset{\sim}{F}_y)\,dy\right] \underset{\sim}{G}\, \underset{\sim}{q} \\
& - \delta \underset{\sim}{q}^T\, c_y\, \theta_o\, \underset{\sim}{G}^T \left[\int_0^a (\underset{\sim}{F}_x^T)\,dx\right] \\
& - \delta \underset{\sim}{q}^T\, c_x\, \theta_o\, \underset{\sim}{G}^T \left[\int_0^b (\underset{\sim}{F}_y^T)\,dy\right]
\end{aligned} \tag{56}$$

As in the one dimensional case already shown, the total variation should be zero to be in equilibrium of this system. Therefore,

$$\delta U_T = \delta U_p + \delta U_w = 0$$

which is from equations (50) and (56),

$$(\delta \underset{\sim}{q}^T)\left\{\underset{\sim}{G}^T[\beta \int_0^a \int_0^b \underset{\sim}{H}^T \underset{\sim}{D}\, \underset{\sim}{H}\, dydx + C_y \int_0^a \underset{\sim x}{F}^T \underset{\sim x}{F}\, dx\right.$$

$$+ C_x \int_0^b \underset{\sim y}{F}^T \underset{\sim y}{F}\, dy]\underset{\sim}{G}\, \underset{\sim}{q} - \underset{\sim}{G}^T[C_y\theta_o \int_0^a \underset{\sim x}{F}^T dx$$

$$\left. + C_x\theta_o \int_0^b \underset{\sim y}{F}^T dy]\right\} = 0 \qquad (57)$$

These are the 16 by 16 simultaneous equations which have to be solved if only one element is concerned as shown in the one dimensional case, with the proper boundary and symmetry conditions. From equation (57), the stiffness matrix for a plate element,

$$\underset{\sim}{k} = \underset{\sim}{G}^T \beta \int_0^a \int_0^b \underset{\sim}{H}^T \underset{\sim}{D}\, \underset{\sim}{H}\, dydx\, \underset{\sim}{G} \qquad (58a)$$

The additional stiffness matrix due to welding along the x-direction of the plate,

$$\underset{\sim wx}{k} = \underset{\sim}{G}^T C_y \int_0^b \underset{\sim x}{F}^T \underset{\sim x}{F}\, dx\, \underset{\sim}{G} \qquad (58b)$$

The additional stiffness matrix due to welding along the y-direction of the plate,

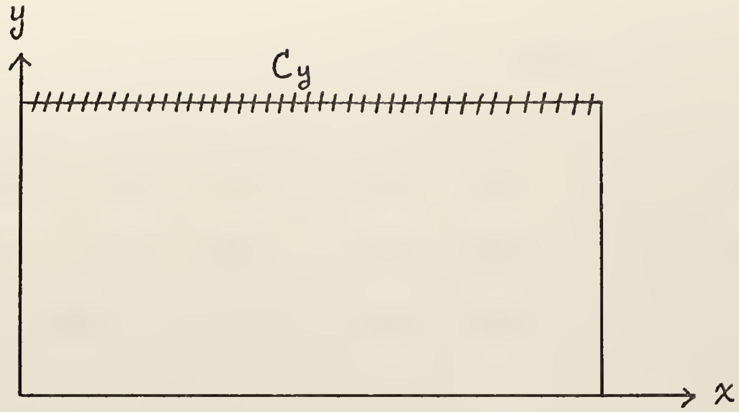
$$\underset{\sim wy}{k} = \underset{\sim}{G}^T C_x \int_0^b \underset{\sim y}{F}^T \underset{\sim y}{F}\, dy\, \underset{\sim}{G} \qquad (58c)$$

The equivalent load matrix due to welding along the x-direction of the plate,

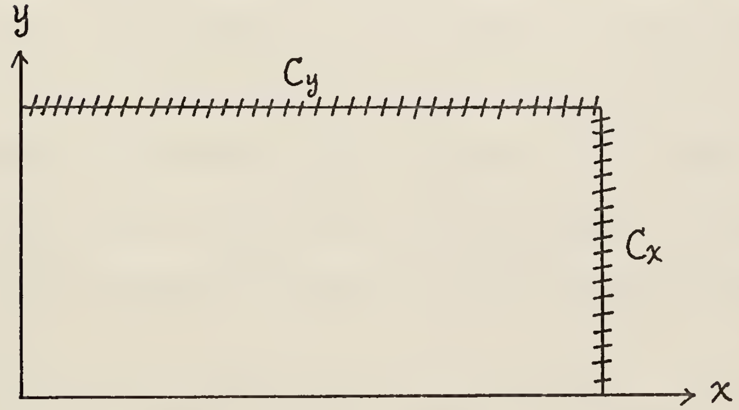
$$\underset{\sim x}{Q} = \underset{\sim}{G}^T C_y\theta_o \int_0^a \underset{\sim x}{F}^T dx \qquad (58d)$$

The equivalent load matrix due to welding along the y-direction of the plate,

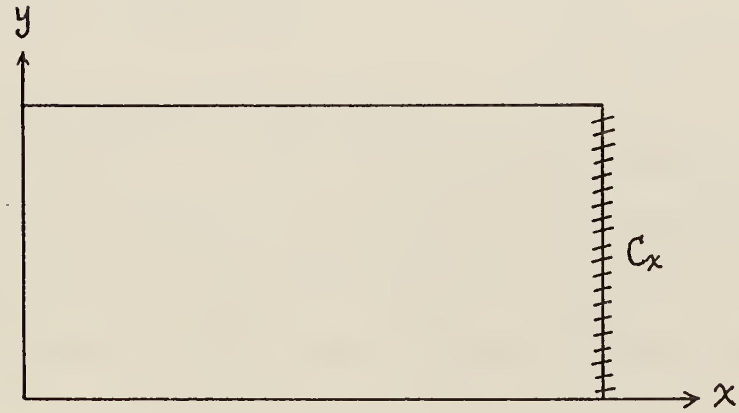
$$\underset{\sim y}{Q} = \underset{\sim}{G}^T C_x\theta_o \int_0^b \underset{\sim y}{F}^T dy \qquad (58e)$$



(a)



(b)



(c)

Figure 6 Various Finite Element Having Welded Edge

2. Assembling Procedure

For all the finite elements except those having the welded edge, the stiffness matrix necessary to be assembled is the expression given by equation (58b), and for the finite elements having welded edges, three cases are involved. The first case is elements having welded edges along the x-direction as in Figure 6b; the second case is along the y-direction, as in Figure 6c; and the third case is along x- and y-directions as in Figure 6b. The assembling procedure is the expression of equation (57) for the overall plate which is to be solved, which is described in FEABL user's manual.[13] Also, from the symmetry condition, which will be described in the following section, only one-quarter part of the plate is actually being considered.

Example of the assembling procedures for the two finite elements is given in Appendix B.

3. Symmetry and Boundary Conditions

After assembling, relevant symmetry and boundary conditions have to be applied to solve the simultaneous equations, as shown in the one dimensional case. Along the welded edge of x and y, the simply supported edge condition requires that all the displacements at that nodal point have to be zero, and for the x-directed edge, the angular changes in the x-direction are not allowed, In just the same way, for y-directed edge, the angular changes in y are not allowed. The symmetry condition requires that the first derivative

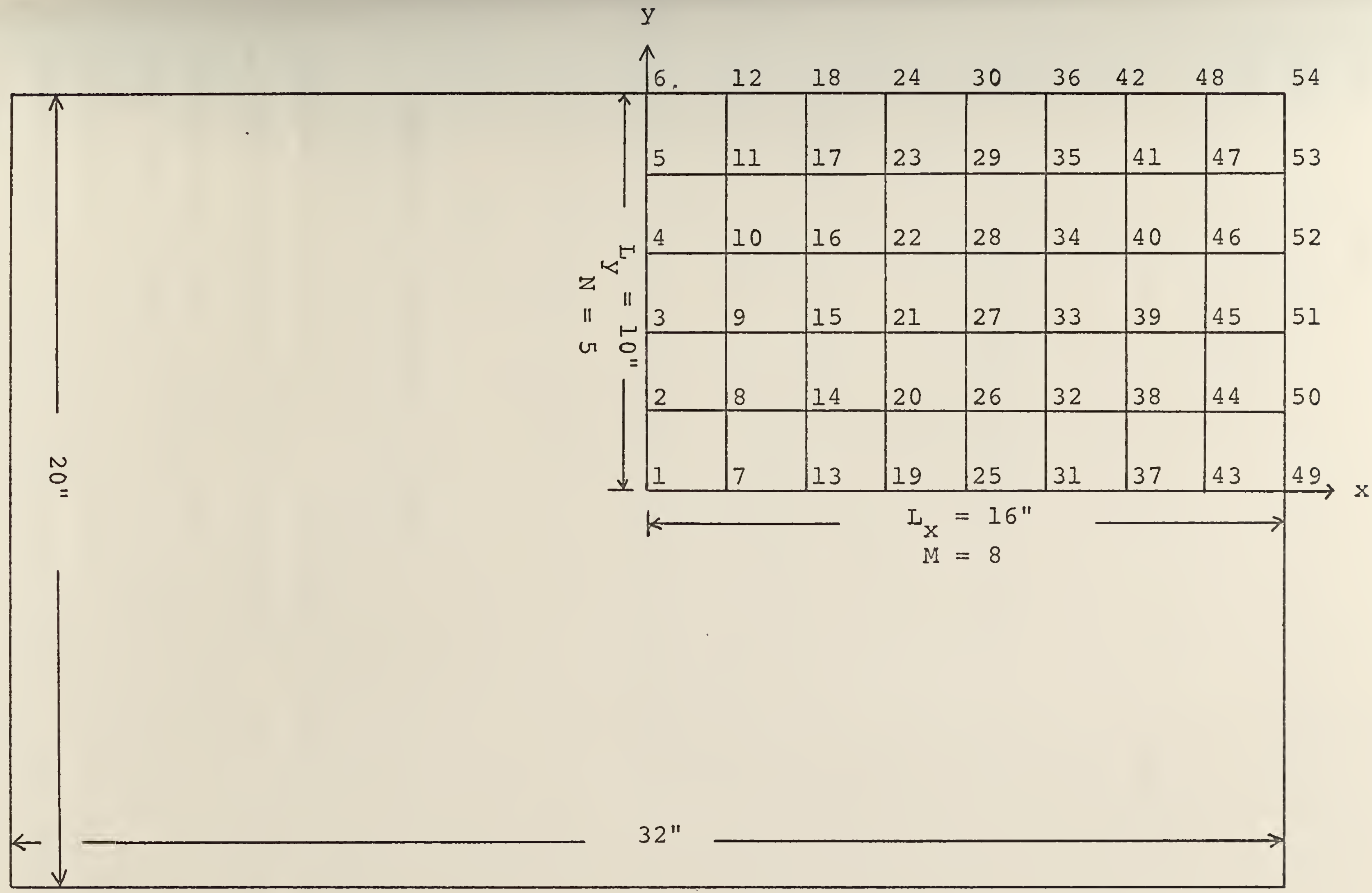


Figure 7 Dimensions and Element Meshes of One-quarter of the Plate for Computer Input

along the symmetric axis should be zero. For example, the symmetry and boundary conditions for the case given by Figure 7 are as follows:

Boundary conditions:

At node 6, 12, 18, 24, 30, 36, 42, 48, 54

$w = 0$

$w_x = 0$

At node 49, 50, 51, 52, 53, 54

$w = 0$

$w_y = 0$

Symmetry conditions:

At node 1, 2, 3, 4, 5, 6

$w_y = 0$

At node 1, 7, 13, 19, 25, 31, 37, 43

$w_x = 0$

At node 1, 6, 49

$w_{xy} = 0.$

4. Numerical Integration Method

Usually in the finite element analysis, the integrations involved are carried out by the numerical methods, which can be easily calculated using the computer. For the present analysis, the nine Gaussian points are chosen for the rectangular finite element,[14,18] as shown in Figure 8c. In actual integration, it is very convenient to use the normalized coordinate system, as will be described below. From Figure 8a and 8b, normalized coordinate system can be defined as

$$x = \frac{a}{2}(X + 1) \qquad (59a)$$

$$y = \frac{b}{2}(Y + 1) \qquad (59b)$$

and

$$dx = \frac{a}{2}\, dX \qquad (60a)$$

$$dy = \frac{b}{2}\, dY \qquad (60b)$$

Therefore, the arbitrary integration

$$f(a,b) = \int_0^a \int_0^b P(x,y)\,dxdy \qquad (61)$$

becomes in the normalized coordinates,

$$f(a,b) = \frac{ab}{4} \int_{-1}^{1} \int_{-1}^{1} P\left[\frac{a}{2}(X+1),\ \frac{b}{2}(Y+1)\right] \qquad (62)$$

Equation (62) can be integrated by numerically using 9-Gaussian points, which become

$$f(a,b) = \frac{ab}{4} \sum_{i=1}^{3} \sum_{j=1}^{3} W_i W_j\ P(x,y) \qquad (63)$$

where

$$x = \frac{a}{2}(X_i + 1) \qquad (63a)$$

$$y = \frac{b}{2}(Y_j + 1) \qquad (63b)$$

$$W_i:\ \ W_1 = 5/9,\ W_2 = 8/9,\ W_3 = 5/9 \qquad (63c)$$

$$W_j:\ \ W_1 = 5/9,\ W_2 = 8/9,\ W_3 = 5/9 \qquad (63d)$$

$$X_i:\ \ X_1 = -\sqrt{15}/5,\ X_2 = 0,\ X_3 = \sqrt{15}/5 \qquad (63e)$$

$$Y_j:\ \ Y_1 = -\sqrt{15}/5,\ Y_2 = 0,\ Y_3 = \sqrt{15}/5 \qquad (63f)$$

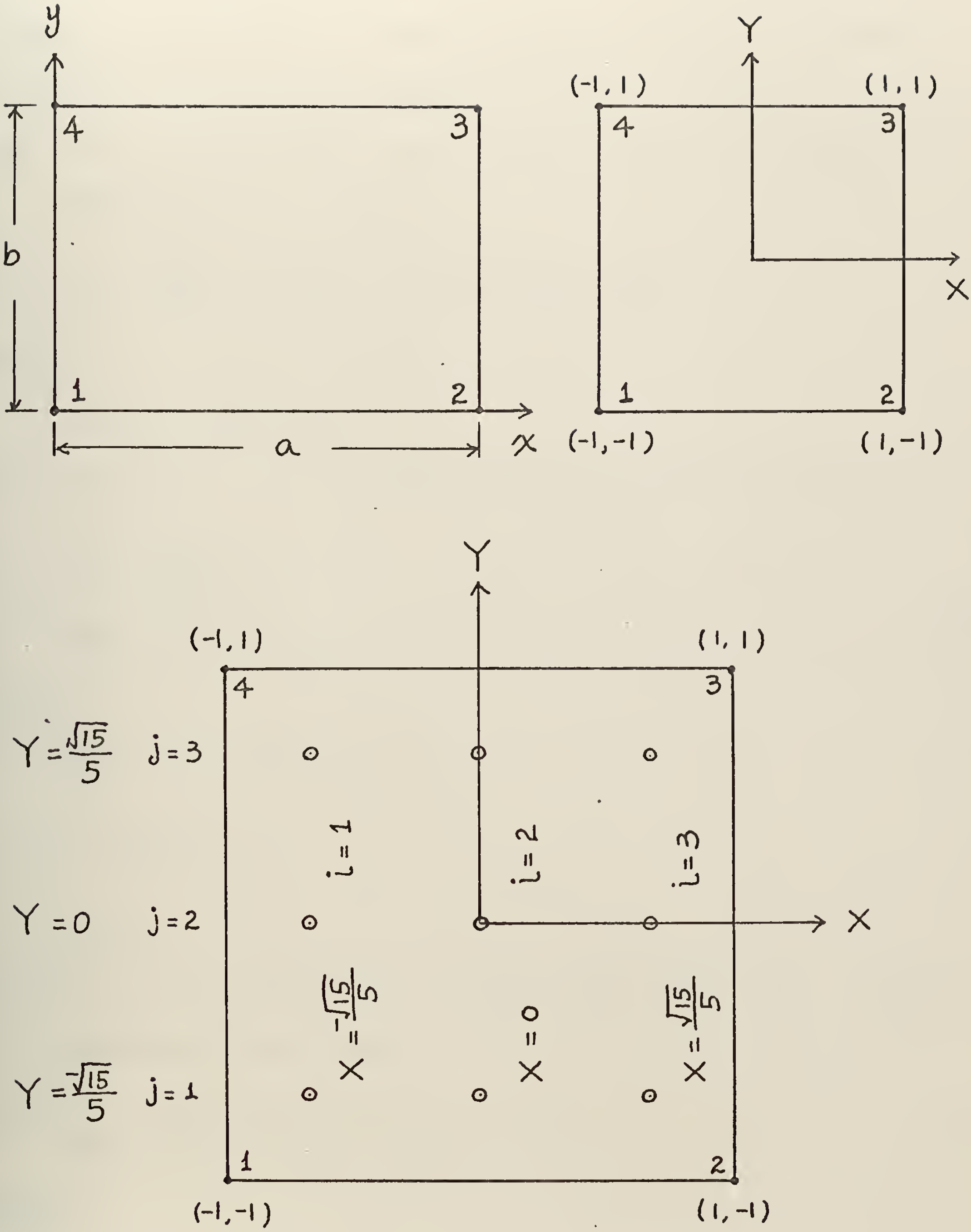


Figure 8 Ordinally and Normalized Coordinates for an Element, and 9 Gausian Points for Numerical Integration

Therefore, using the concept of normalizing and numerical integration described above, the integrations involved in the present analysis such as equation (58a-e) can easily be written, which are programmed in Appendix D.

From equation (58a)

$$\underset{\sim}{k} = \left(\frac{ab\beta}{4}\right) \underset{\sim}{G}^T \left[\sum_{i=1}^{3} \sum_{j=1}^{3} W_i W_j \underset{\sim}{P}(x,y) \right] \underset{\sim}{G} \tag{64}$$

where

$$\underset{\sim}{P}(x,y) = \underset{\sim}{H}^T \underset{\sim}{D} \underset{\sim}{H}$$

x, y, W_i, W_j, X_i and Y_j are defined by equation (63a-f).

From equation (58b),

$$\underset{\sim}{k}_{wx} = \frac{aC_y}{2} \underset{\sim}{G}^T \left[\sum_{i=1}^{3} W_i \underset{\sim}{P}_x(x,y) \right] \underset{\sim}{G} \tag{65}$$

where

$$\underset{\sim}{P}_x(x,y) = \underset{\sim}{F}_x^T \underset{\sim}{F}_x$$

$$x = \frac{a}{2}(X_i + 1)$$

$$y = b$$

W_i and X_i are defined by equation (63c) and (63e), respectively.

From equation (58c),

$$\underset{\sim}{k}_{wy} = \frac{bC_x}{2} \underset{\sim}{G}^T \left[\sum_{j=1}^{3} W_j P_y(x,y) \right] \underset{\sim}{G} \tag{66}$$

where

$$\underset{\sim}{P}_y(x,y) = \underset{\sim}{F}_y^T \underset{\sim}{F}_y$$

$$x = a$$

$$y = \frac{b}{2}(Y_j + 1)$$

W_j and X_j are defined by equations (63d) and (63f), respectively.

From equation (58d),

$$\underset{\sim}{Q}_x = \frac{a\theta_o C_y}{2} \underset{\sim}{G}^T [\sum_{i=1}^{3} W_i \underset{\sim}{P}_{\ell x}(x,y)] \underset{\sim}{G} \tag{67}$$

where

$$\underset{\sim}{P}_{\ell x}(x,y) = \underset{\sim}{F}_x^T$$

$$x = \frac{a}{2}(X_i + 1)$$

$$y = b$$

W_i and X_i are defined by equation (63c) and (63), respectively.

From equation (58e),

$$\underset{\sim}{Q}_y = \frac{b\theta_o C_x}{2} \underset{\sim}{G}^T [\sum_{j=1}^{3} W_j \underset{\sim}{P}_{\ell y}(x,y)] \underset{\sim}{G} \tag{68}$$

where

$$\underset{\sim}{P}_{\ell y}(x,y) = \underset{\sim}{F}_y^T$$

$$x = a$$

$$y = \frac{b}{2}(Y_j + 1)$$

W_j and Y_j are defined by equations (63d) and (63f).

As noticed above, numerical integration using the normalized coordinate can simplified the computing procedure by factoring out a and b.

5. Computer Programs

Computations of the ordinally, additional welding, and equivalent welding load matrix of equation (58a-e) have been programmed by using the numerical integration technique with normalized coordinates as listed in Appendix D. In addition to these, FEABL Programs[13] in assembling of the relevant matrices and solving the simultaneous equations with proper boundary and symmetry conditions are also used and listed in Appendix D.

The Programs listed in Appendix D are only relevant to the panel structure welding with rectangular finite elements. Further, the nodal numbering system should follow the way given by the sample analysis, which means that the numbering should start from origin to y-direction, as shown in Figure 7.

Input data for this analysis are the overall dimensions of one panel structure in the x- and y-direction, thickness of the plate, Young's modulus of elasticity, Poisson's ratio, number of elements divided in x- and y-direction, equivalent welding moment in x- and y-direction, and free joint angular change at the edge. The input data cards are described in Appendix C.

Outputs are the displacements, angular changes in x- and y-directions, and the twisting angles at each nodal point, as shown in Appendix E.

One thing to be noticed here is that for a quarter part of the plate, as shown in Figure 9, the proper boundary and symmetry conditions at the nodal points are to be generated

by subroutine HOLD given in Appendix D; therefore, as long as boundary conditions of simply supported and symmetry conditions as described are concerned, it is not necessary to be considered in the input. Therefore, no matter how the number of finite elements are to be chosen, the only input data will be as described in equation (69a-h). Furthermore, to use this program, the coordinate and a quarter part of the plate should be chosen as the same way as shown in Figure 9.

The units used in the sample program are pounds, inches, and radians. As results, for the output data, the deflections are in inches, the first derivatives (angular changes) are in radians, and the second derivatives with respect to x and y (twisting angles) are also in radians.

Program language used is FORTRAN IV, and the computations are carried out by an IBM 370/155 system.

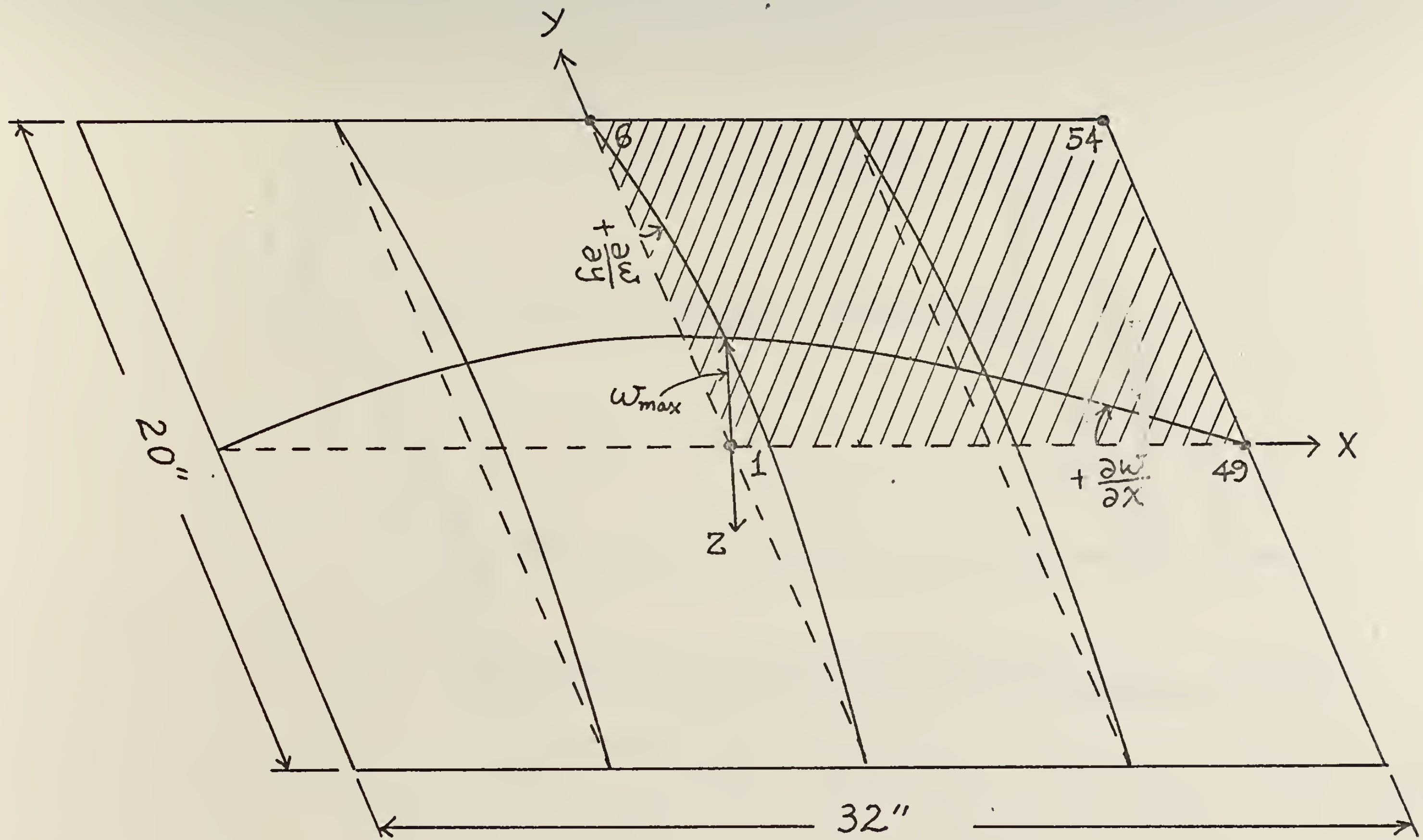


Figure 9 Global Coordinate System and Shape of Welded Deflection

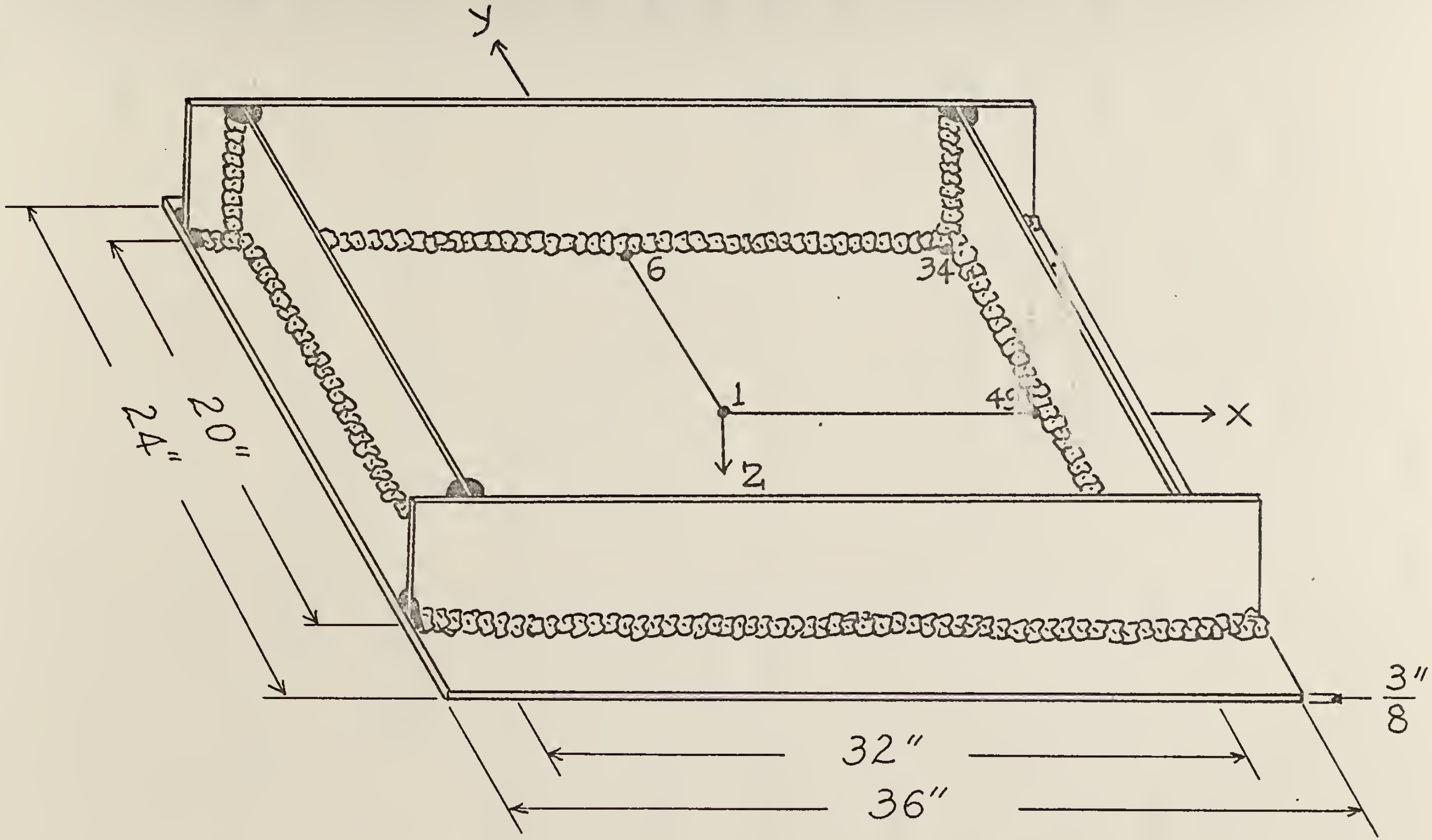


Figure 10 A Panel Structure Analyzed by Experiment and Finite Element Method

III RESULTS

As computer inputs, one steel panel structural dimension, as shown in Figure 10, for which the experiment conducted and weight of consumed welding rods are given, 3 gr/cm.,[4,19] are used to predict the deformations, angular changes, and twisting angles at the nodal points. Here, the value of the one dimensional free joint angular change along the welded edge, θ_o, has been read from the results of Figure 3 and the equivalent constrained welding moment, C, is being evaluated using equation (8). The numerical values of the input data are as follows:

Half-length of the plate in the x- and y-directions, respectively, L_x = 16 inches and L_y = 10 inches.

Number of finite elements in the x- and y-directions, respectively, M = 8 and N = 5.

Equivalent constrained welding moment from equation (8)

$$C = \frac{t^4}{1 + W/5} = \frac{(25.4 \times 3/8)^4}{1 + 3/5} = 5200 \ \frac{\text{kg-mm}}{\text{mm rad.}}$$

where W is given by 3 gr/cm from experiment.

Therefore,

$$C = 1.13 \times 10^4 \text{ lb-in/in rad.}$$

Poisson's ratio:

$$\nu = 0.3$$

Young's modulus:

$$E = 30 \times 10^6 \text{ psi.}$$

Thickness of the plate:

$$t = 3/8 \text{ inch.} \qquad (69\text{ a-h})$$

The results of the analysis using the above data are listed in Appendix E, and these results are plotted in the Figures 11 and 12 to be compared with experimental results. As can be noticed from Figures 11 and 12, the finite element results are lower value than the experiments, and the maximum deflection at the midpoint of the plate which is the most important deflection in reality is only a half of that from the experiment.

To visualize the behavior of the deformations as functions of θ_o and C, series of computations are conducted with different combinations of θ_o and C as shown in Table 1. Yet remaining, the rest of the input data are unchanged, and the maximum deflections at nodal point 1 are plotted in terms of θ_o and C as in Figures 13 and 14, and the maximum angular changes at node 6 and 49 are plotted in the Figures 15 and 16. Only the maximum deflections at node 1, and the maximum angular changes at node 6 and 49 from the results of series computations are tabulated in Tables 2 and 3.

Finally, computed results using $\theta_o = 55 \times 10^{-3}$ and $C = 4.2 \times 10^4$ are listed in Appendix E, and also plotted in Figures 11 and 12 to compare with the experiment. As can be seen from Figures 11 and 12, these results are very close to the experiment and the reason of choosing the above θ_o and C will be discussed in the following section.

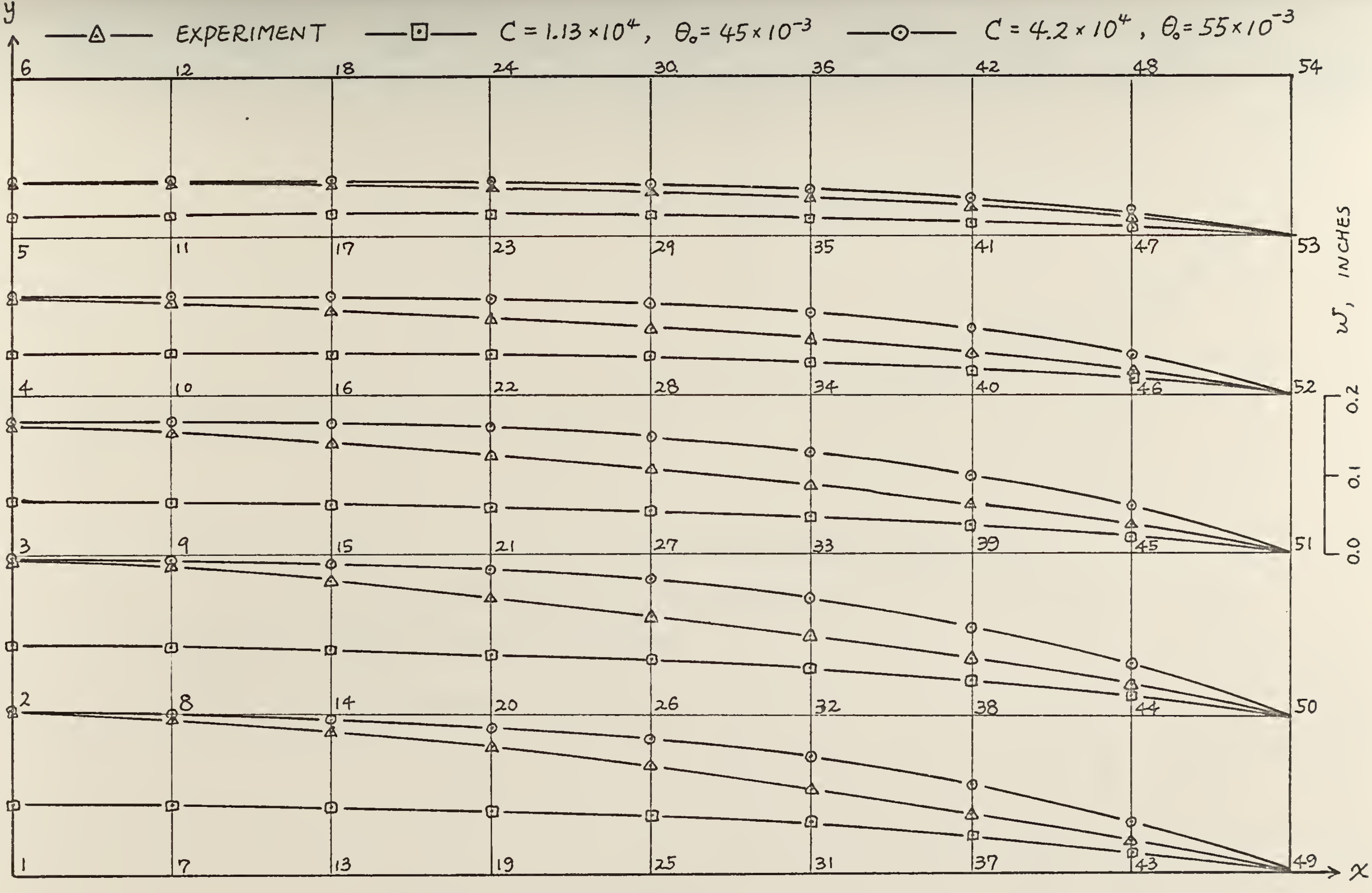


Figure 11 Deflection Comparison with Experiment and Finite Element Analysis

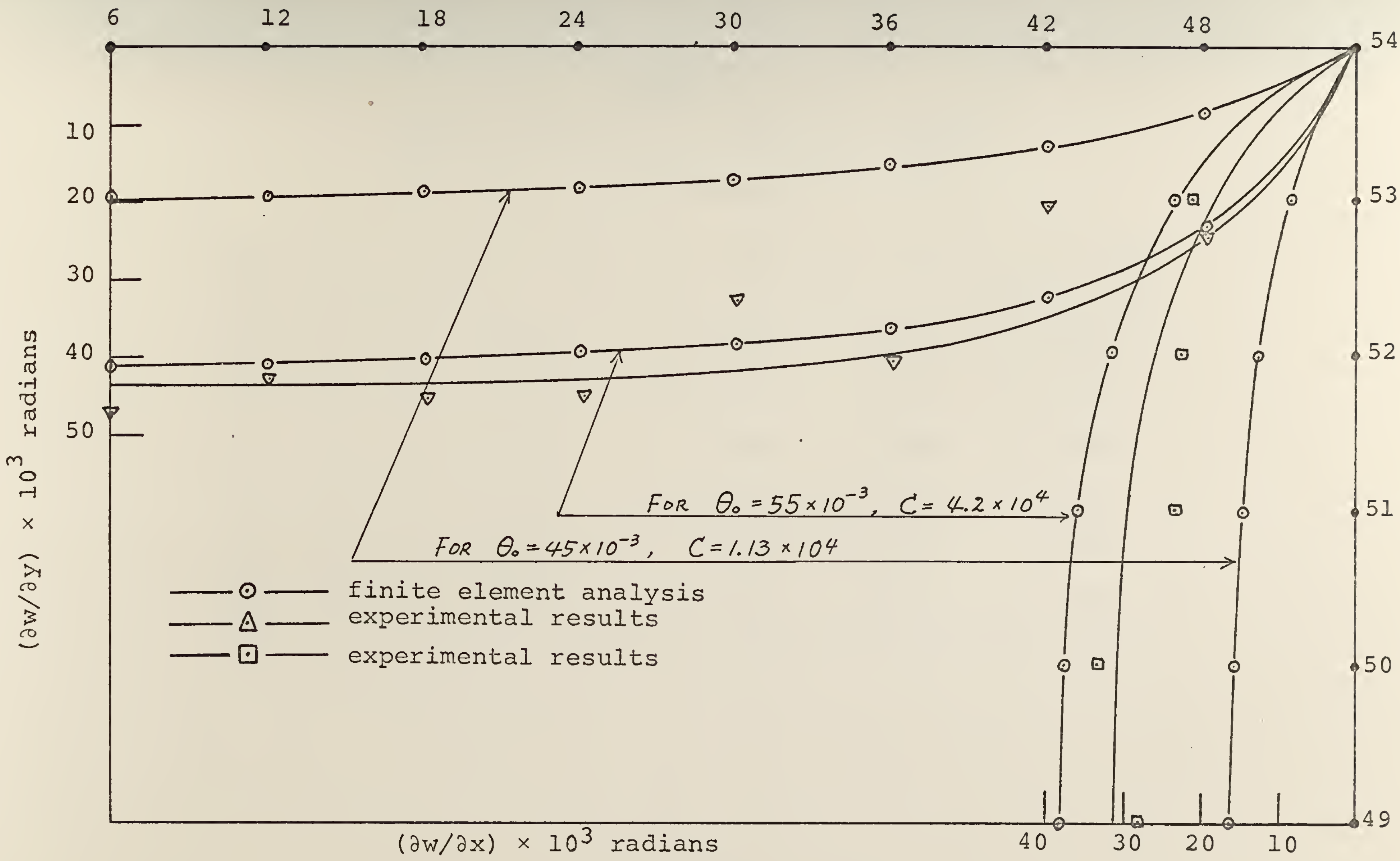


Figure 12 Angular Changes along the Welded Boundary Compared with Experiment and Finite Element Analysis

Table 1

Maximum Deflection at Node 1 in Inches

with Different Combinations of C and θ_o

θ_o \ C	1.13×10^4	4×10^4	8×10^4	11×10^4
45×10^{-3}	0.092	0.166	0.196	0.206
55×10^{-3}	0.112	0.203	0.240	0.252
65×10^{-3}	0.133	0.240	0.283	0.298
75×10^{-3}	0.153	0.277	0.327	0.343
85×10^{-3}	0.174	0.314	0.370	0.389
95×10^{-3}	0.194	0.351	0.414	0.435

Table 2

Maximum Angular Changes ($\partial w/\partial y$) at Node 6

in 10^3 radians with Different Combinations of C and θ_o

θ_o \ C	1.13×10^4	4×10^4	8×10^4	11×10^4
45×10^{-3}	18.86	32.28	38.67	40.33
55×10^{-3}	23.05	40.68	47.26	49.30
65×10^{-3}	27.23	48.07	55.85	58.26
75×10^{-3}	31.43	55.47	64.45	67.23
85×10^{-3}	35.62	62.86	73.04	76.29
95×10^{-3}	39.81	70.26	81.63	85.16

Table 3

Maximum Angular Changes $(\partial w/\partial x)$ at Node 49 in 10^3 radians with Different Combinations of C and θ_o

θ_o \ C	1.13×10^4	4×10^4	8×10^4	11×10^4
45×10^{-3}	16.59	31.00	37.18	39.22
55×10^{-3}	20.28	37.90	45.45	47.94
65×10^{-3}	23.97	44.79	53.71	56.66
75×10^{-3}	27.66	51.68	61.97	65.38
85×10^{-3}	31.35	58.57	70.24	74.09
95×10^{-3}	35.04	65.46	78.50	82.81

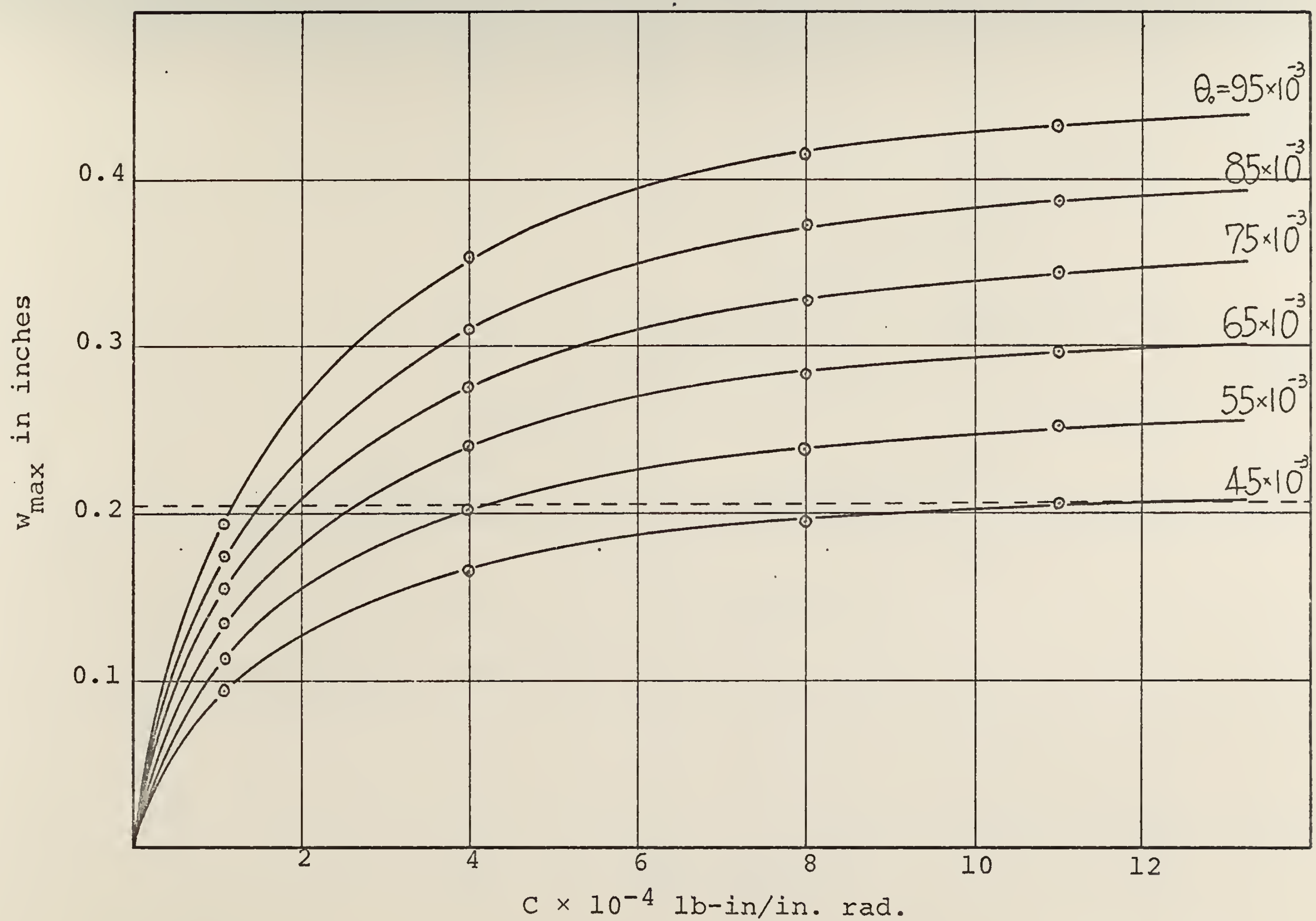


Figure 13 w_{max} at Node 1 versus C with Different θ_o, Dashed Line Represents Experimental Value.

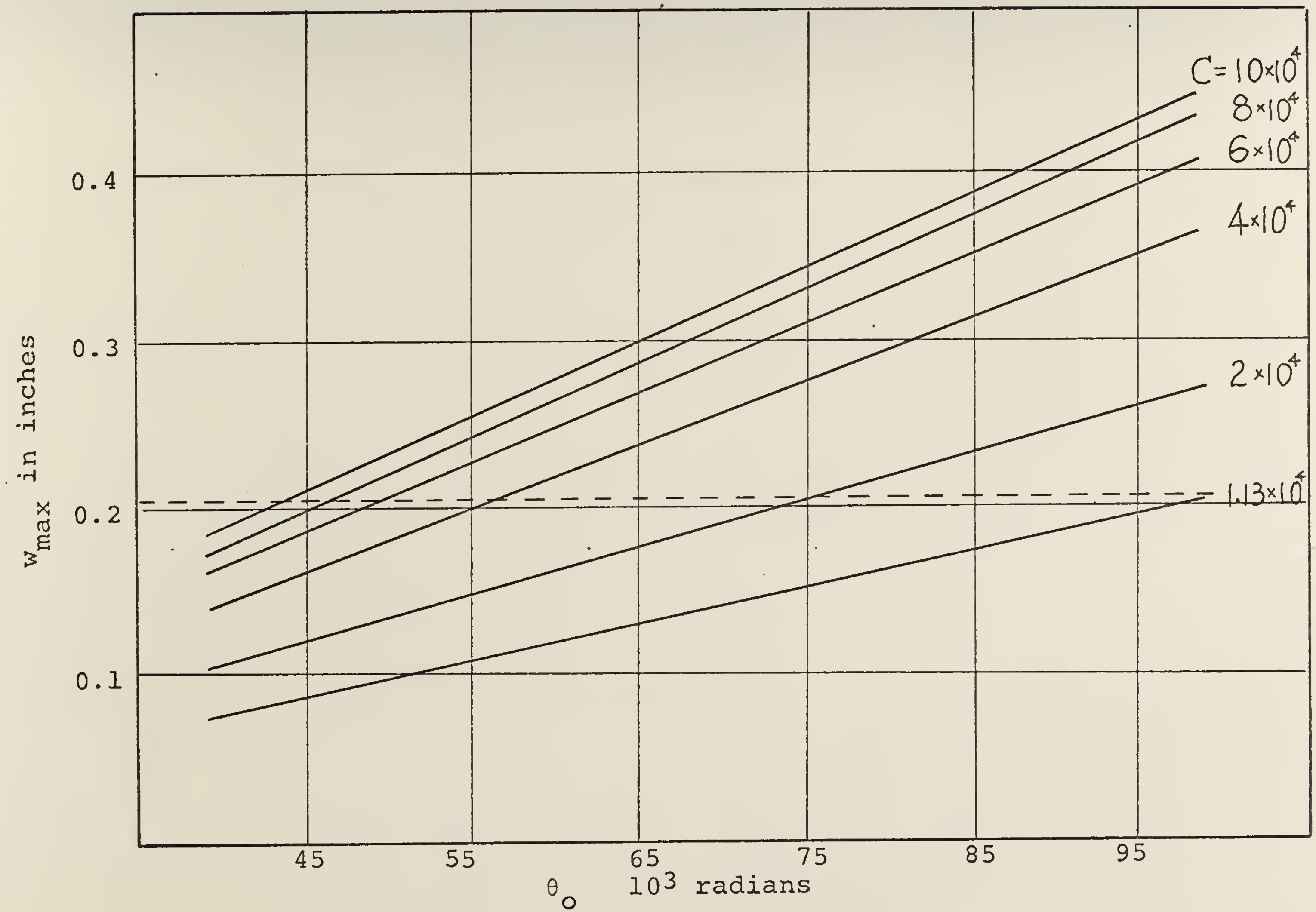


Figure 14 w_{max} at Node 1 versus θ_o with Different C, Dashed Line Represents Experimental Value.

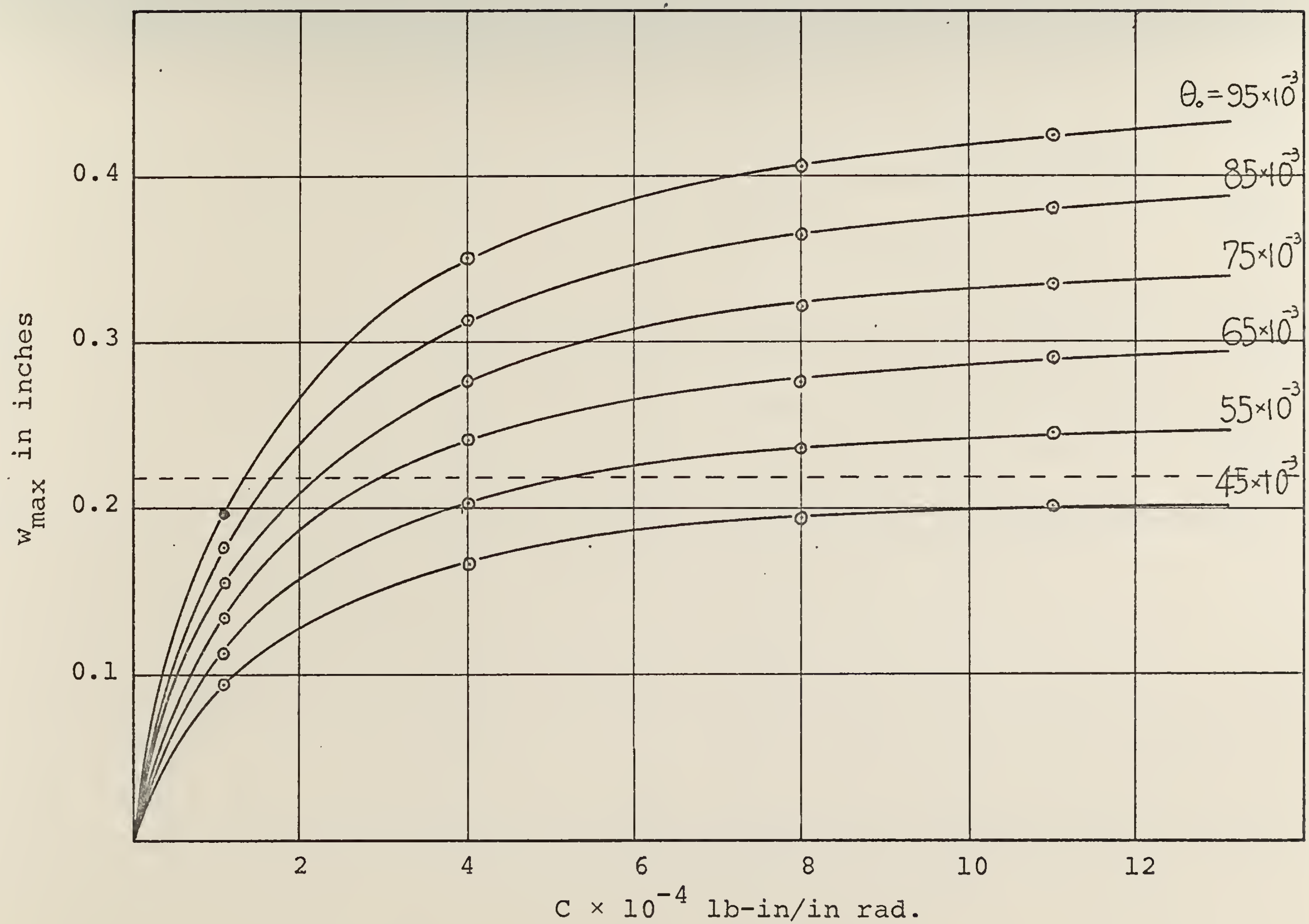


Figure 15 $(\partial w/\partial y)_{max}$ at Node 6, Dashed Line Represents Experimental Value

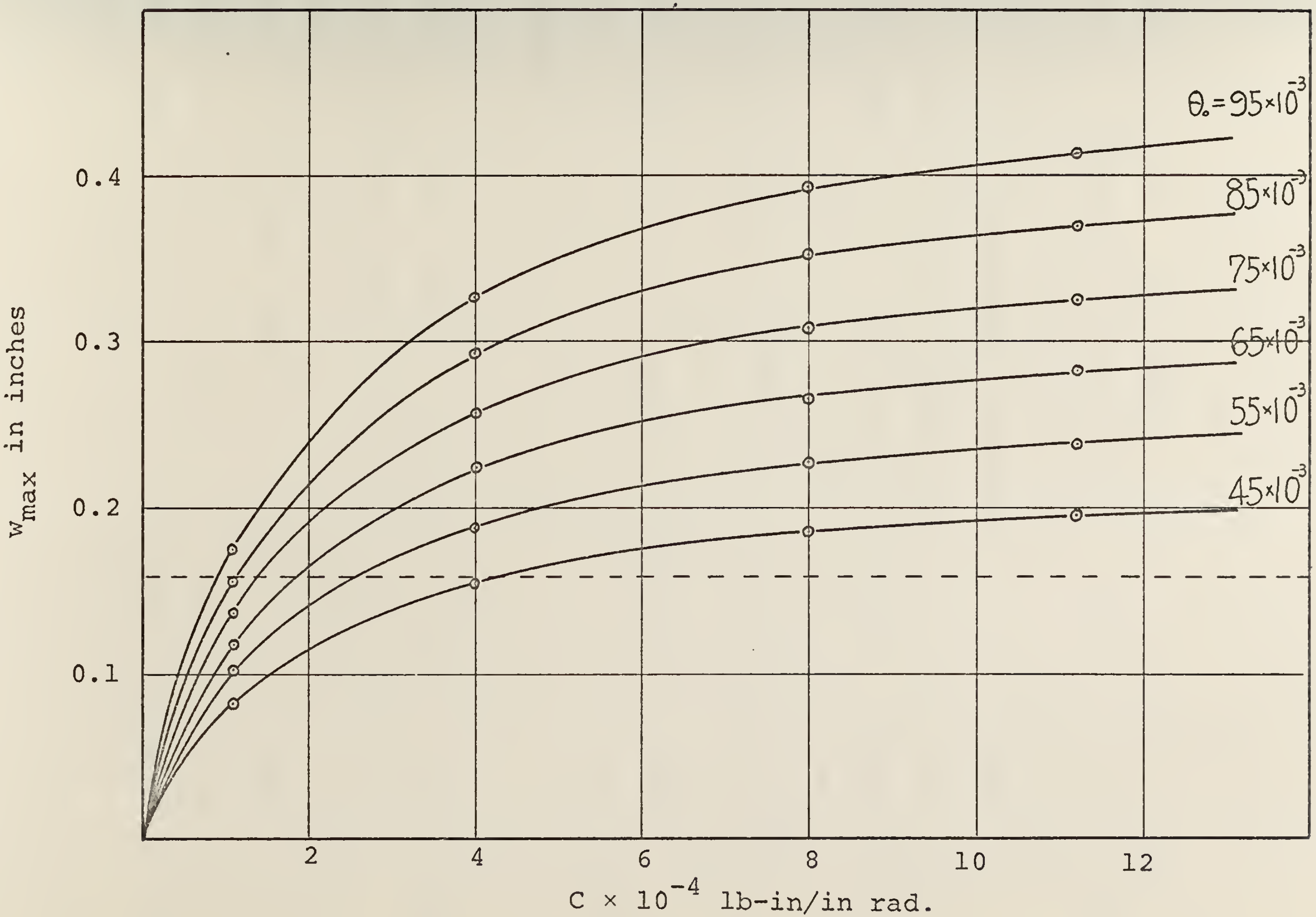


Figure 16 $(\partial w/\partial x)_{max}$ at Node 49, Dashed Line Represents Experimental Value

IV DISCUSSION OF RESULTS

As has been mentioned in the previous section, the results using $C = 1.13 \times 10^4$ and $\theta_o = 45 \times 10^{-3}$ are only a half of the experimentally measured values as shown in Figures 11 and 12. The reasons of this difference are not yet known. These may result from the assumption of the elastic behavior of the welded deformation, or from the uncertainty of the experimental measurement itself because the deflections and angular changes which are only of the order of magnitude of 10^{-1} inches and 10^{-2} radians, respectively. Yet, other possibilities are from the value of θ_o and C.

To investigate the behavior of the deformation with respect to θ_o and C, series of computations have been conducted. The plotting of maximum deflections at node 1 versus C with changing θ_o as in Figure 13 shows some insight of deflection behavior with varying C. In here, it is obvious that the deflections are more rapidly changed with changing θ_o than C changes, which means that the deflections are more strongly dependent function of θ_o rather than C. Furthermore, the deflection reaches rapidly to an asymptotic value as C increases, which can be seen clearly from the one dimensional deflection, equation (41) where the term $2EI/\ell c$ becomes small compared to 1 as C increases; therefore, in the limit case, the deflection can be expressed only by $w = \ell\theta_o/4$.

This behavior of the deflection with changing C are clearly shown in Figure 13. Also from Figure 14, in which the maximum deflections are plotted with respect to θ_o, it is obvious that the deflections are rapidly increased as θ_o increases.

If assumed that the difference between experiment and computed results are caused from the possibility of different values of θ_o and C, then the question is how to choose θ_o and C, such that the computed maximum deflection becomes the same to that from the experiment. From Figures 12 and 13, the combinations of θ_o and C to satisfy the given value of maximum deflection from the experiment can be predicted. However, the best combination is not known, but for the known experiment value of maximum deflection it may be safe to choose θ_o and C such that the changes of both θ_o and C from the calculated value be minimized. In other words, superposing Figures 13 and 14 as shown in Figure 17, the point which gives the same reading of θ_o and C in both ways as shown by the small circle, which is read as $\theta_o = 55 \times 10^{-3}$ and $C = 4.2 \times 10^4$, will be the optimal combination to satisfy the given maximum deflection. The comparison of the results using this θ_o and C to the experiments, as in Figures 11 and 12, shows that this approach gives reasonably good results.

As has been seen from above, there exists many uncertainties for the two dimensional welding analysis, but the welding deformation phenomena can be predicted with a reasonable degree of accuracy by the finite element analysis.

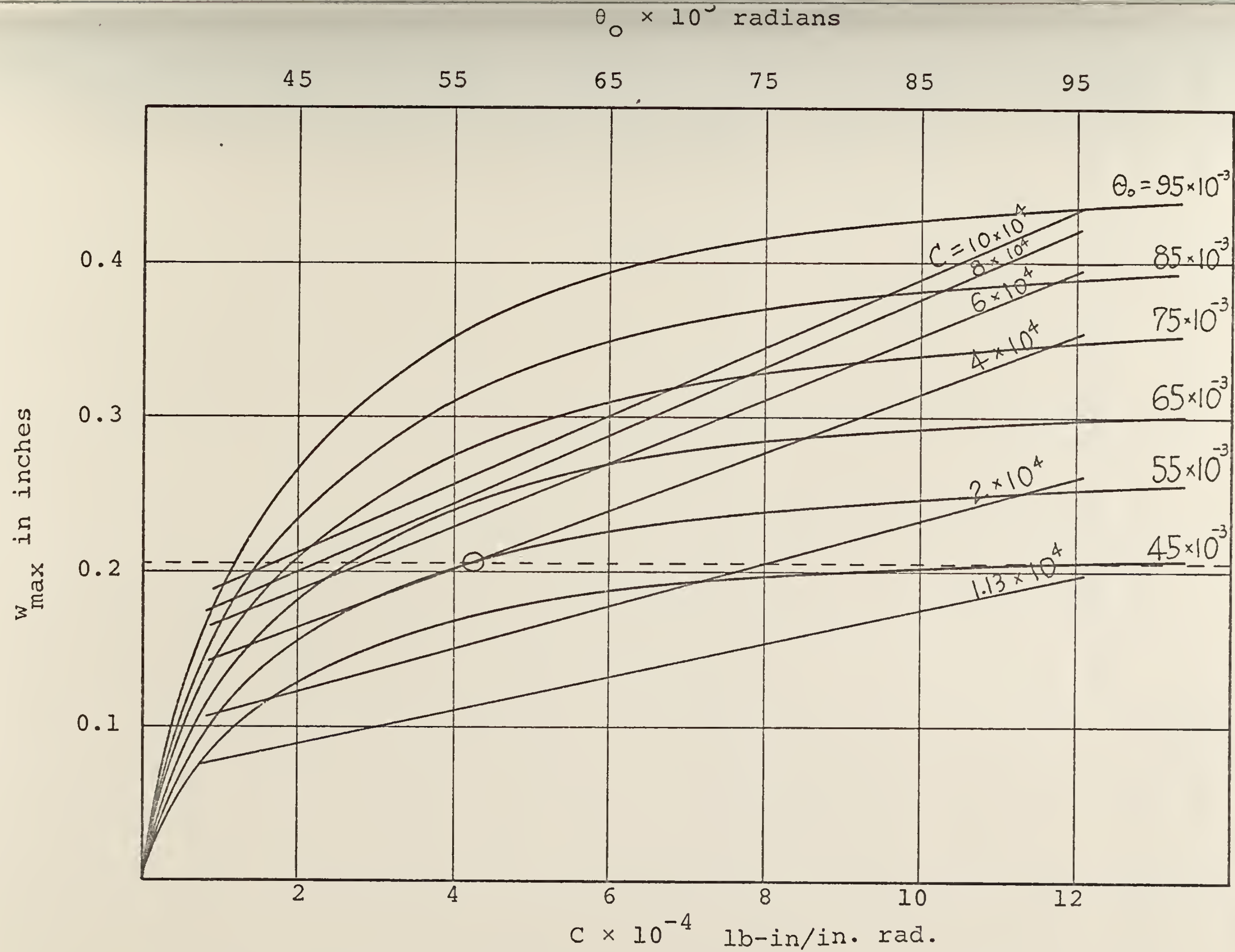


Figure 17 w_{max} at node 1 after superposition of Figures 13 and 14, dashed line represents experimental value.

Furthermore, the empirical formula of C given by equation (8) may not be simply applied to the two dimensional case, and also does the value of θ_o. If it is so, measuring only the maximum deflection in future experiments, the best combination of θ_o and C can be predicted as discussed.

V CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

From the discussion of the previous section, it can be concluded as:

1. The finite element analysis for the two dimensional welding deformations which are assumed to be elastic deformation gives reasonable prediction of deflections.
2. The searching technique of optimal combination of θ_o and C for a given deflection can be used as shown already, if there exists the uncertainty of θ_o and C.
3. In the future experiments, this analysis gives the order of magnitude feeling of the deflection measured after welding. It is extremely difficult to measure correct deflections in experiments because the order of magnitude to be measured is very small; therefore, the inherent uncertainties always exist.
4. The magnitude of importance of the welding deformation variables such as θ_o and C are clearly seen, therefore, more care should be concentrated in measuring θ_o rather than C.
5. Simple empirical formula for C given by equation (8) may not cover the wide range of the values of C; furthermore, as can be derived from equation (41) it may be logical to express C in terms of maximum deflection, free joint angular change, and rigidity,

such as:

$$C = \theta_o/4w_{max} - 2EI/\ell \tag{70}$$

in which θ_o is a function of a given welding condition and w_{max} is the constrained maximum deflection at midpoint of the span for the same condition of the welding as of θ_o. Further, the elastic rigidity and plate dimension terms are expressed by the term of $2EI/\ell$.

B. Recommendations

In this analysis, the assumption used is the elastic behavior of deformations. As stated in the conclusion, it is still believed reasonable to use the elastic theory, but for more accurate prediction, local plastic deformation phenomena should be included in the analysis.

If the elastic assumption is to be used, more careful analysis to evaluate the value of θ_o and C are recommended in future research. Also, a more rational approach to express C is recommended.

In future research, it is recommended that series of experiments be conducted to measure the maximum plate deformation only, and express C and θ_o in terms of welding and plate dimension. In this way, the computer inputs are always available for any prediction of deformation for the two dimensional panel structures.

APPENDIX A

Definitions of Matrices

$$
\underset{\sim}{C} = \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \\ c_{13} \\ c_{14} \\ c_{15} \\ c_{16} \end{Bmatrix} \qquad \underset{\sim}{q} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \\ q_{11} \\ q_{12} \\ q_{13} \\ q_{14} \\ q_{15} \\ q_{16} \end{Bmatrix} = \begin{Bmatrix} w_1 \\ w_{x_1} \\ w_{y_1} \\ w_{xy_1} \\ w_2 \\ w_{x_2} \\ w_{y_2} \\ w_{xy_2} \\ w_3 \\ w_{x_3} \\ w_{y_3} \\ w_{xy_3} \\ w_4 \\ w_{x_4} \\ w_{y_4} \\ w_{xy_4} \end{Bmatrix}
$$

$$\underset{\sim}{T} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & a & 0 & 0 & a^2 & 0 & a^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 2a & 0 & 3a^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & a & 0 & 0 & 0 & a^2 & 0 & 0 & a^3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 2a & 0 & 0 & 3a^2 & 0 & 0 & 0 & 0 & 0 \\
1 & a & b & ab & a^2 & b^2 & a^3 & a^2b & ab^2 & b^3 & a^3b & a^2b^2 & ab^3 & a^3b^2 & a^2b^3 & a^3b^3 \\
0 & 1 & 0 & b & 2a & 0 & 3a^2 & 2ab & b^2 & 0 & 3a^2b & 2ab^2 & b^3 & 3a^2b^2 & 2ab^3 & 3a^2b^3 \\
0 & 0 & 1 & a & 0 & 2b & 0 & a^2 & 2ab & 3b^2 & a^3 & 2a^2b & 3ab^2 & 2a^3b & 3a^2b^2 & 3a^3b^2 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 2a & 2b & 0 & 3a^2 & 4ab & 3b^2 & 6a^2b & 6ab^2 & 9a^2b^2 \\
1 & 0 & b & 0 & 0 & b^2 & 0 & 0 & 0 & b^3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & b & 0 & 0 & 0 & 0 & b^2 & 0 & 0 & 0 & b^3 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 2b & 0 & 0 & 0 & 3b^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2b & 0 & 0 & 0 & 3b^2 & 0 & 0 & 0
\end{bmatrix}$$

$$\underset{\sim}{H} = \begin{pmatrix} 0 & 0 & 0 & 0 & 2 & 0 & 6x & 2y & 0 & 0 & 6xy & 2y^2 & 0 & 6xy^2 & 2y^3 & 6xy^3 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2x & 6y & 0 & 2x^2 & 6xy & 2x^3 & 6x^2y & 6x^3y \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 4x & 4y & 0 & 6x^2 & 8xy & 6y^2 & 12x^2y & 12xy^2 & 18x^2y^2 \end{pmatrix}$$

$$\underset{\sim}{F}_x = [0 \quad 0 \quad 1 \quad x \quad 0 \quad 2y \quad 0 \quad x^2 \quad 2xy \quad 3y^2 \quad x^3 \quad 2x^2y \quad 3xy^2 \quad 2x^3y \quad 3x^2y^2 \quad 3x^3y^2]$$

$$\underset{\sim}{F}_y = [0 \quad 1 \quad 0 \quad y \quad 2x \quad 0 \quad 3x^2 \quad 2xy \quad y^2 \quad 0 \quad 3x^2y \quad 2xy^2 \quad y^3 \quad 3x^2y^2 \quad 2xy^3 \quad 3x^2y^3]$$

APPENDIX B

Example of Assembling

As an example, the assembling procedure for a quarter part of the plate divided by two finite element shown in Figure 18a will be discussed.

For the element 1, as shown in Figure 18b, the stiffness matrix in a local coordinate can be written from equation (58a, b),

$$\underset{\sim}{k}_{t_1} = \underset{\sim}{k} + \underset{\sim}{k}_{w_x} \qquad \text{(A-1)}$$

where $\underset{\sim}{k}_{t_1}$ is 16 by 16 symmetric matrix. And from equation (58d), the load matrix for the element 1 is,

$$\underset{\sim}{Q}_{t_1} = \underset{\sim}{Q}_x \qquad \text{(A-2)}$$

where $\underset{\sim}{Q}_{t_1}$ is 16 by 1 matrix. By the same way for element 2, the stiffness and load matrix can be written using equation (58a, b, c) and (58d, e), respectively,

$$\underset{\sim}{k}_{t_2} = \underset{\sim}{k} + \underset{\sim}{k}_{wx} + \underset{\sim}{k}_{wy} \qquad \text{(A-3)}$$

$$\underset{\sim}{Q}_{t_2} = \underset{\sim}{Q}_x + \underset{\sim}{Q}_y \qquad \text{(A-4)}$$

where $\underset{\sim}{k}_{t_2}$ is 16 by 16 symmetric matrix and $\underset{\sim}{Q}_{t_2}$ is 16 by 1 matrix.

At every nodal point there are four unknowns (w, w_x, w_y, w_{xy}), therefore, after assembling element 1 and 2, the equations have to be solved with proper boundary and symmetry conditions are 24 by 24 simultaneous equations, which is in the matrix form:

$$\delta \underset{\sim}{q}^T [\underset{\sim}{K} \, \underset{\sim}{q} - \underset{\sim}{Q}] = 0 \qquad \text{(A-5)}$$

where the matrix $\underset{\sim}{K}$ is 24 by 24 symmetric matrix, $\underset{\sim}{Q}$ is 24 by 1, q is 24 by 1, and $\underset{\sim}{q}^T$ is 1 by 24 matrix.

All the quantities in equation (A-5) are defined by the global coordinate, therefore, the assembling procedure is to locate the quantities defined by equation (A-1, A-2, A-3, A-4) to the proper position in equation (A-5). This operation can easily be established comparing each nodal points numbered in local coordinates to that of global coordinates:

<u>Element 1, nodal points</u>		<u>Element 2, nodal points</u>	
<u>Coordinates</u>		<u>Coordinates</u>	
local	global	local	global
1	1	1	3
2	3	2	5
3	4	3	6
4	2	4	4

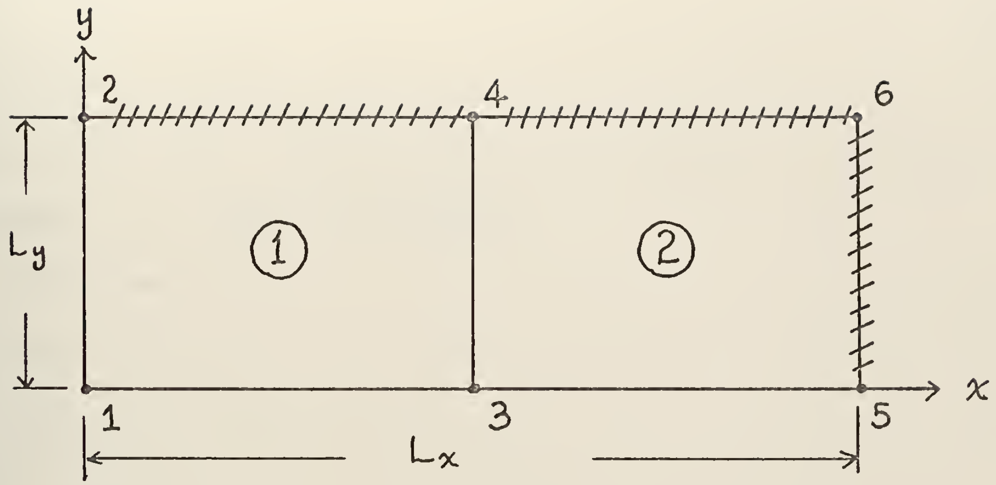
And noticing that numbering of the generalized displacement, q_i, follows the relation,

$$i = 4m - g \qquad \text{(A-6)}$$

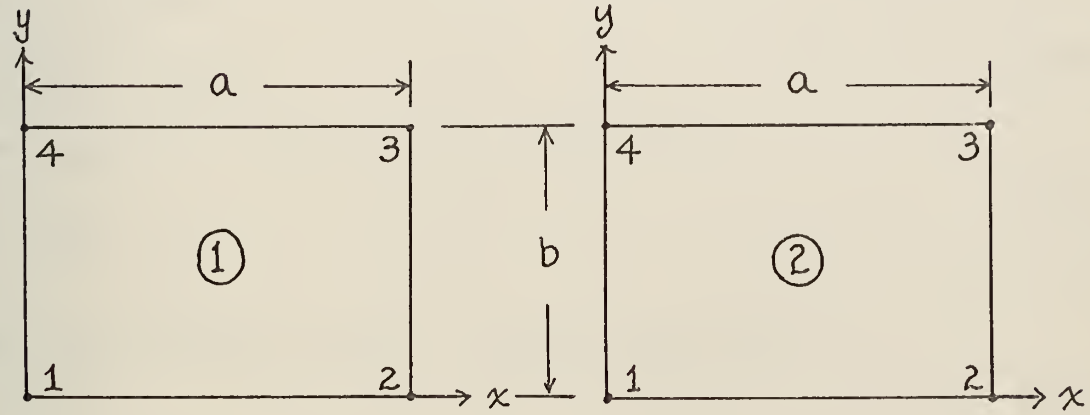
$$j = 4n - h \qquad \text{(A-7)}$$

where i and j represent the index for generalized displacements, m and n for nodal numbers, g and h are set to be 3 for w, 2 for w_x, 1 for w_y, and 0 for w_{xy}.

For example, one of the stiffness matrix elements at nodal point 4 in global coordinate can be calculated.



a. Nodal points numbering in global coordinate



b. Nodal points numbering in local coordinate

Figure 18 Elements and Nodal Points Numbering in Global and Local Coordinate for Sample Assembling

From equation (A-6) and (A-7),

$$i = 16 - g \tag{A-8}$$

$$j = 16 - h \tag{A-9}$$

But the same node 4 becomes node 3 at element 1 and node 4 at element 2 in local coordinate.

For element 1,

$$(i)_1 = 12 - g \tag{A-10}$$

$$(j)_1 = 12 - h \tag{A-11}$$

and for element 2,

$$(i)_2 = 16 - g \tag{A-12}$$

$$(j)_2 = 16 - h \tag{A-13}$$

Therefore, if $g = 1$ and $h = 2$, then from equation (A-8), (A-9), and (A-5),

$$K_{ij} = K_{15,14} \tag{A-14}$$

From equations (A-10), (A-11), and (A-1)

$$k_{t_1\,11,10} \tag{A-15}$$

and from equations (A-12), (A-13), and (A-3),

$$k_{t_2\,15,14} \tag{A-16}$$

Therefore, element of the stiffness matrix in global coordinate becomes,

$$K_{15,14} = k_{t_1\,11,10} + k_{t_2\,15,14} \tag{A-17}$$

All other stiffness matrix element can be expressed by the same manner described above, and the programs of assembling the stiffness matrix are available in FEABL.[(13)]

APPENDIX C

Description of Input Data

Input data for the programs listed in Appendix D are: C_x, C_y, θ_o, D, t, E, L_x, L_y, M and N as described in section III. In addition to these, control card for calculation of different cases in each job with changing above input variables are considered. By using this calculation control input card in each job, series of calculation with changing some of the input variables can be calculated, and the deformation behavior with respect to these changing variables can easily be visualized by plotting the deformation versus these variables.

Detailed input data card arrangement and format are listed in Table 4.

Table 4

Input Data Layout Form

Card Column	Input Format	Program Symbol	Definition and Unit	Sample Input
Card Number 1				
1-5	I5	NCASE	Number of cases involved in each job	14
Card Number 2				
1-15	E15.7	LX	Length of 1/4 plate in x, inches	16.0E+00
16-30	E15.7	LY	Length of 1/4 plate in y	10.0E+00
31-35	I5	M	Number of elements in x	8
36-40	I5	N	Number of elements in y	5
Card Number 3				
1-12	E12.7	CX	Equivalent constrained welding moment as in Figure 6, lb-in/in rad.	4.20E+04
13-24	E12.7	CY	Equivalent constrained welding moment as in Figure 6, lb-in/in rad.	4.20E+04
25-36	E12.7	THETAZ	Free joint angular change, θ_o, radians	55.0E-03
37-48	E12.7	GNU	Poisson's ratio, D	0.30E+00
49-60	E12.7	THK	Plate thickness, t, inches	0.375E+00
61-72	E12.7	E	Young's modulus, E, lb/in.^{2}	30.0E+06

APPENDIX D

Listing of Programs

```
C     DAWPS PROGRAM
C      DISTORTION ANALYSIS OF WELDED PANEL STRUCTURE
C     FORMULATED BY D. SHIN
C     PROGRAMMED BY SUSAN E. FRENCH.....ASRL AT MIT FEB. 1972
C             THIS PROGRAM USES FEABL, FINITE ELEMENT ANALYSIS BASIC
C     LIBRARY, WRITTEN BY DR. OSCAR ORRINGER OF THE AERONAUTICS AND
C     ASTRONAUTICS DEPT. OF MIT
      REAL*4 LX,LY
      DIMENSION REAL(8000),INTGR(8000)
      DIMENSION D(3,3),G(16,16),ELK1(16,16),ELK2(16,16),ELK3(16,16),
     1          ELK4(16,16),Q1(16),Q2(16),Q3(16),Q4(16)
      COMMON /IO/ KREAD,KWRITE,KPUNCH
      COMMON /SIZE/ NET, NDT
      COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK
      COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK
      COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETAZ,ABBET,HALFA,HALFB,ASQ,ACU
     1B,BSQ,BCUB,CXB2,CYA2,BTHCX,ATHCY
      COMMON /GAUSS/ WBAR(3),XBAR(3),YBAR(3)
      COMMON /MESH/ NDE,NNT,LTYPE(200)
      EQUIVALENCE (REAL(1), INTGR(1))
C     DEFINE LDIM=DIMENSION OF REAL AND INTGR
      LDIM=8000
C     DEFINE READ , WRITE AND PUNCH DATA SET REFERENCE NUMBERS
      KREAD=5
      KWRITE=6
      KPUNCH=7
C     READ ALL INPUT DATA
      READ(KREAD,900) NCASE
  900 FORMAT(I5)
      DO 100 NRUN=1,NCASE
      READ (KREAD,1000) LX,LY,M,N,CX,CY,THETAZ,GNU,THK,E
 1000 FORMAT(2E15.7,2I5/6E12.7)
C
C     LX=LENGTH IN X DIRECTION, LY=LENGTH IN Y DIRECTION, M= NO. MESHES IN X
C     DIRECTION, N=NO. OF MESHES IN Y DIRECTION,CX AND CY ARE CONSTANTS,
C     THETAZ=THETA SUB ZERO=AN ANGLE IN RADIANS, GNU=NU=POISSONS RATIO,
```

```
C     THK=SMALL T=THICKNESS, E= YOUNGS MODULUS
C
C     NET=NO. OF ELEMENTS (TOTAL) IN MESH,NDE=NO. OF DEGREES OF FREEDOM
C     PER MESH ELEMENT,NNT=NO. OF NODES (TOTAL) IN MESH,NDT=NO. OF
C     DEGREES OF FREEDOM (TOTAL) IN MESH,NCON= NO. OF CONSTRAINTS
C     (BOUNDARY CONDITIONS) APPLIED TO TOTAL MESH
      NET=N*M
      NDE=16
      NNT=(N+1)*(M+1)
      NDT=4*NNT
      MASTRL=NET*(1+NDE)
      NCON=4*(N+M+1)
C     CALCULATE SOME CONSTANTS
      BETA=E*THK**3/(12.*(1.-GNU*GNU))
      A=LX/FLOAT(M)
      B=LY/FLOAT(N)
      HALFA=.5*A
      HALFB=.5*B
      ASQ=A*A
      ACUB=ASQ*A
      BSQ=B*B
      BCUB=BSQ*B
      ABBET=A*B*BETA/4.
      CXB2=.50*CX*B
      CYA2=.50*CY*A
      BTHCX=.5*B*THETAZ*CX
      ATHCY=.5*A*THETAZ*CY
C     CALCULATE T MATRIX, D MATRIX AND (INVERSE OF T) = G MATRIX
      CALL TDGMAT(GNU,D,G)
C     CALCULATE DATA IN BEGIN AND END LABELLED COMMON
      CALL SETUP(LDIM,NCON,MASTRL,REAL,INTGR)
C     CALCULATE AND STORE MASTER VECTOR DATA RELATING ELEMENTS AND
C     THE DEGREE OF FREEDOM NUMBERS (GLOBAL) AT EACH CORNER
      CALL NODDOF(REAL,INTGR)
C     FILL IN KOUNT AND LNZE SECTIONS OF INTGR AND CALCULATE LK
      CALL ORK(LDIM,REAL,INTGR)
```

```
C     DEFINE CONSTANTS NEEDED FOR GAUSSIAN QUADRATURE INTEGRATION
      WBAR(1)=.5555556
      WBAR(2)=.8888889
      WBAR(3)=.5555556
      XBAR(1)=-.7745967
      XBAR(2)=0.
      XBAR(3)=+.7745967
      YBAR(1)=-.7745967
      YBAR(2)=0.
      YBAR(3)=+.7745967
C     CALCULATE ELEMENT STIFFNESS MATRIX FOR  NO WELD
      CALL STIF1(D,G,ELK1,Q1)
      CALL STIFY(G,ELK2,Q2)
      CALL STIFX(G,ELK3,Q3)
      DO 40 KK=1,NDE
      Q2(KK)=Q2(KK)+Q1(KK)
      Q4(KK)=Q2(KK)+Q3(KK)
      Q3(KK)=Q3(KK)+Q1(KK)
      DO 30 LL=1,NDE
      ELK2(KK,LL)=ELK1(KK,LL)+ELK2(KK,LL)
      ELK4(KK,LL)=ELK2(KK,LL)+ELK3(KK,LL)
      ELK3(KK,LL)=ELK3(KK,LL)+ELK1(KK,LL)
   30 CONTINUE
   40 CONTINUE
C     ASSEMBLE TOTAL STIFFNESS MATRIX AND TOTAL Q VECTOR
      DO 60 J=1,4
      DO 50 LNUM=1,NET
      IF(LTYPE(LNUM).NE.J) GO TO 50
      GO TO (42,44,46,48), J
   42 CALL ASEMBL(LNUM,NDE,ELK1,Q1,REAL,INTGR)
      GO TO 50
   44 CALL ASEMBL(LNUM,NDE,ELK2,Q2,REAL,INTGR)
      GO TO 50
   46 CALL ASEMBL(LNUM,NDE,ELK3,Q3,REAL,INTGR)
      GO TO 50
   48 CALL ASEMBL(LNUM,NDE,ELK4,Q4,REAL,INTGR)
```

```
   50  CONTINUE
   60  CONTINUE
C      FILL IN CONSTRAINT VECTOR FOR ENTIRE STRUCTURE
       CALL HOLD(N,M,ICON,NCON,IQ,INTGR,REAL)
C      APPLY CONSTRAINTS TO ASSEMBLED K MATRIX
       CALL BCON(REAL,INTGR)
       CALL FACTSD(REAL,INTGR)
       CALL SIMULQ(ENERGY,REAL,INTGR)
C      PRINT ALL INPUT DATA
       WRITE(KWRITE,2000) LX,LY,M,N,CX,CY,THETAZ,GNU,THK,E
 2000 FORMAT(1H1/1H0/'0DISTORTION ANALYSIS OF WELDED PANEL STRUCTURE....
     1BY D. SHIN'/4H0LX=,E12.5,5H, LY=,E12.5,23H, M=NO. ELEMENTS X DIR=,
     2I3/20H0NO. ELEMENTS Y DIR=,I3,5H, CX=,E12.5,5H, CY=,E12.5/9H0THETA
     3 0=,E12.5,25H RADIANS, POISSONS RATIO=,E12.5/11H0THICKNESS=,E12.5,
     417H, YOUNGS MODULUS=,E12.5)
      WRITE(KWRITE,2120)
 2120 FORMAT(5H0NODE,7X,1HW,12X,2HDW,12X,2HDW,8X,10HD( DW/DY )/1H+,23X,4
     1H____,10X,4H____,6X,12H____________/1H ,24X,2HDX,12X,2HDY,12X,2HDX
     2)
      ISTART=IQ-4
      DO 80 NO=1,NNT
      ISTART=ISTART+4
      IEND=ISTART+3
      WRITE (KWRITE,2130) NO,(REAL(J),J=ISTART,IEND)
 2130 FORMAT(I4,4E14.5)
 80   CONTINUE
 100  CONTINUE
      STOP
      END
```

```
      SUBROUTINE TDGMAT(GNU,D,G)
C     CALCULATE T, D AND G=INVERSE OF D MATRICES
      COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETAZ,ABBET,HALFA,HALFB,ASQ,ACU
     1B,BSQ,BCJB,CXB2,CYA2,BTHCX,ATHCY
      COMMON /IO/ KREAD,KWRITE,KPUNCH
      DIMENSION D(3,3),G(16,16),T(16,16),LL(16),MM(16),ALIN(256)
      CALL ERASE (T,256)
      T(1,1)=1.
      T(2,2)=1.
      T(3,3)=1.
      T(4,4)=1.
      T(5,1)=1.
      T(5,2)=A
      T(5,5)=ASQ
      T(5,7)=ACUB
      T(6,2)=1.
      T(6,5)=2.*A
      T(6,7)=3.*ASQ
      T(7,3)=1.
      T(7,4)=A
      T(7,8)=ASQ
      T(7,11)=ACUB
      T(8,4)=1.
      T(8,8)=T(6,5)
      T(8,11)=T(6,7)
      T(9,1)=1.
      T(9,2)=A
      T(9,3)=B
      T(9,4)=B*A
      T(9,5)=ASQ
      T(9,6)=BSQ
      T(9,7)=ACUB
      T(9,8)=ASQ*B
      T(9,9)=A*BSQ
      T(9,10)=BCUB
      T(9,11)=ACUB*B
```

```
T(9,12)=ASQ*BSQ
T(9,13)=A*BCUB
T(9,14)=ACUB*BSQ
T(9,15)=ASQ*BCUB
T(9,16)=ACUB*BCUB
T(10,2)=1.
T(10,4)=B
T(10,5)=T(6,5)
T(10,7)=T(8,11)
T(10,8)=2.*A*B
T(10,9)=BSQ
T(10,11)=3.*T(9,8)
T(10,12)=2.*T(9,9)
T(10,13)=BCUB
T(10,14)=3.*T(9,12)
T(10,15)=2.*T(9,13)
T(10,16)=3.*T(9,15)
T(11,3)=1.
T(11,4)=A
T(11,6)=2.*B
T(11,8)=ASQ
T(11,9)=T(10,8)
T(11,10)=3.*BSQ
T(11,11)=ACUB
T(11,12)=2.*T(9,8)
T(11,13)=3.*T(9,9)
T(11,14)=2.*T(9,11)
T(11,15)=T(10,14)
T(11,16)=3.*T(9,14)
T(12,4)=1.
T(12,8)=T(10,5)
T(12,9)=T(11,6)
T(12,11)=T(10,7)
T(12,12)=4.*T(9,4)
T(12,13)=T(11,10)
T(12,14)=6.*T(9,8)
```

```
      T(12,15)=6.*T(9,9)
      T(12,16)=9.*T(9,12)
      T(13,1)=1.
      T(13,3)=B
      T(13,6)=BSQ
      T(13,10)=BCUB
      T(14,2)=1.
      T(14,4)=B
      T(14,9)=BSQ
      T(14,13)=BCUB
      T(15,3)=1.
      T(15,6)=T(11,6)
      T(15,10)=T(11,10)
      T(16,4)=1.
      T(16,9)=T(15,6)
      T(16,13)=T(15,10)
      D(1,1)=1.
      D(2,1)=GNU
      D(3,1)=0.
      D(1,2)=GNU
      D(2,2)=1.
      D(3,2)=0.
      D(1,3)=0.
      D(2,3)=0.
      D(3,3)=.5*(1.-GNU)
C     CONVERT T MATRIX TO A VECTOR FOR INPUT TO SUBROUTINE MINV OF SSP
      L=0
      DO 30 J=1,16
      DO 30 I=1,16
      L=L+1
   30 ALIN(L)=T(I,J)
C     INVERT MATRIX T
      CALL MINV (ALIN,16,DET,LL,MM)
C     CHECK THAT DET=DETERMINANT IS NON-ZERO
      IF (DET.NE.0) GO TO 40
      WRITE(KWRITE,3020)
```

```
3020 FORMAT(68HOT MATRIX IS SINGULAR. EXECUTION IS TERMINATED IN SUBROU
    1TINE TDGMAT./1H1)
      STOP
C     CONVERT ALIN TO A 16X16 MATRIX=THE G MATRIX
 40   L=0
      DO 50 J=1,16
      DO 50 I=1,16
      L=L+1
 50   G(I,J)=ALIN(L)
      RETURN
      END
```

```
      SUBROUTINE HOLD (N,M,ICON,NCON,IQ,INTGR,REAL)
C     THIS SUBROUTINE GENERATES THE BOUNDARY CONDITIONS ASSUMING
C     THAT ONLY THE UPPER RIGHT CORNER OF A RECTANGULAR PANEL IS BEING
C     CONSIDERED. THE ENTIRE PANEL IS ASSUMED TO BE WELDED AROUND THE EDGES
      DIMENSION INTGR(2),REAL(2)
      JAY=ICON-1
      JDOF1=-3
      MP1=M+1
      NP1=N+1
      DO 120 JJ=1,MP1
      DO 100 II=1,NP1
C     DEFINE THE GLOBAL DOF NOS. FOR THE AT Y(II),X(JJ)
      JDOF1=JDOF1+4
      JDOF2=JDOF1+1
      JDOF3=JDOF2+1
      JDOF4=JDOF3+1
C     TEST TO SEE IF (X,Y) IS AN EDGE POINT
      IF(JJ.NE.MP1.AND.JJ.NE.1.AND.II.NE.NP1.AND.II.NE.1) GO TO 100
C     IS AN EDGE POINT
      IF(JJ.EQ.1) GO TO 10
      GO TO 40
C     LEFT EDGE
   10 JAY=JAY+1
      INTGR(JAY)=JDOF2
      JAY=JAY+1
      INTGR(JAY)=JDOF4
      IF(II.EQ.1) GO TO 20
      IF(II.EQ.NP1) GO TO 30
      GO TO 100
C     LOWER LEFT CORNER
   20 JAY=JAY+1
      INTGR(JAY)=JDOF3
      GO TO 100
C     UPPER LEFT CORNER
   30 JAY=JAY+1
      INTGR(JAY)=JDOF1
```

```
      GO TO 100
C     NOT ON LEFT EDGE
   40 IF(JJ.EQ.MP1) GO TO 70
C     NOT ON RIGHT EDGE OR LEFT EDGE
      IF(II.EQ.1) GO TO 50
      IF(II.EQ.NP1) GO TO 60
      GO TO 100
C     ON LOWER EDGE
   50 JAY=JAY+1
      INTGR(JAY)=JDOF3
      JAY=JAY+1
      INTGR(JAY)=JDOF4
      GO TO 100
C     ON TOP EDGE
   60 JAY=JAY+1
      INTGR(JAY)=JDOF1
      JAY=JAY+1
      INTGR(JAY)=JDOF2
      GO TO 100
C     ON RIGHT EDGE
   70 JAY=JAY+1
      INTGR(JAY)=JDOF1
      JAY=JAY+1
      INTGR(JAY)=JDOF3
      IF(II.EQ.1) GO TO 80
      IF(II.EQ.NP1) GO TO 90
      GO TO 100
C     ON LOWER RIGHT CORNER
   80 JAY=JAY+1
      INTGR(JAY)=JDOF4
      GO TO 100
C     ON UPPER RIGHT CORNER
   90 JAY=JAY+1
      INTGR(JAY)=JDOF2
  100 CONTINUE
  120 CONTINUE
```

```
      NCON=JAY
      DO 130  I=1,NCON
      KSUB=ICON-1+I
      MSUB=INTGP(KSUB)
      LSUB=IQ-1+MSUB
      REAL(LSUB)=0.
130   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE NODDOF(REAL,INTGR)
      DIMENSION REAL(2),INTGR(2)
      COMMON /SIZE/ NET, NDT
      COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK
      COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK
      COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETAZ,ABBET,HALFA,HALFB,ASQ,ACU
     1B,BSQ,BCUB,CXB2,CYA2,BTHCX,ATHCY
      COMMON /MESH/ NDF,NNT,LTYPE(200)
C     THIS SUBROUTINE CALCULATES THE GLOBAL DOF NUMBERS
      KSUB=IMASTR-1
      N4P1=4*(N+1)
      KPUT=IMASTR+NET
      DO 40 JJ=1,M
      DO 30 II=1,N
      LNUM=II+(JJ-1)*N
C     INTERNAL (NO EDGE WELDED)
      IF(II.LT.N.AND.JJ.LT.M) LTYPE(LNUM)=1
C     ALONG RIGHT EDGE (ONLY RIGHT EDGE WELDED)
      IF(II.NE.N.AND.JJ.EQ.M) LTYPE(LNUM)=2
C     ALONG TOP EDGE (ONLY TOP EDGE WELDED)
      IF(II.EQ.N.AND.JJ.LT.M)LTYPE(LNUM)=3
C     UPPER RIGHT CORNER (TWO EDGES WELDED)
      IF(II.EQ.N.AND.JJ.EQ.M) LTYPE(LNUM)=4
      KSUB=KSUB+1
      INTGR(KSUB)=KPUT
      KDOF=4*(LNUM+JJ-1)-3
      DO 20 KK=1,4
C     LOWER LEFT CORNER OF MESH
      K=KK-1
      KSUB1=KPUT+K
      INTGR(KSUB1)=KDOF+K
C     LOWER RIGHT CORNER OF MESH
      KSUB2=KSUB1
      KSUB1=KSUB1+4
      INTGR(KSUB1)=INTGR(KSUB2)+N4P1
C     UPPER RIGHT CORNER OF MESH
```

```
      KSUB2=KSUB1
      KSUB1=KSUB1+4
      INTGR(KSUB1)=INTGR(KSUB2)+4
C     UPPER LEFT CORNER OF MESH
      KSUB1=KSUB1++
      KSUB2=KSUB2-4
      INTGR(KSUB1)=INTGR(KSUB2)+4
   20 CONTINUE
      KPUT=KPUT+16
   30 CONTINUE
   40 CONTINUE
      LAST=KSUB1
      IF(LMASTR.GT.LAST) GO TO 50
      RETURN
   50 KSUB1=LAST+1
      DO 60 I=KSUB1,LMASTR
   60 INTGR(I)=0
      RETURN
      END
```

```
      SUBROUTINE STIF1(D,G,STIFFM,Q)
C     THIS SUBROUTINE CALCULATES THE ELEMENT STIFFNESS MATRIX FOR NO WELD
      DIMENSION H(3,16),P(16,16),HTDH(16,16), STIFFM(16,16),Q(16),WF(16)
      DIMENSION D(3,3),G(16,16)
      COMMON /IO/ KREAD,KWRITE,KPUNCH
      COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETAZ,ABBET,HALFA,HALFB,ASQ,ACU
     1B,BSQ,BCUB,CX32,CY32,BTHCX,ATHCY
      COMMON /GAUSS/ WBAR(3),XBAR(3),YBAR(3)
      CALL ERASE (H,48,STIFFM,256,P,256,Q,16)
      H(1,5)=2.
      H(2,6)=2.
      H(3,4)=2.
      DO 48 I=1,3
      XBI=XBAR(I)
      WI=WBAR(I)
      X=HALFA*(XBI+1)
      H(1, 7)=6.*X
      H(2,9 )=2.*X
      H(2,12)=H(2,9)*X
      H(2,14)=H(2,12)*X
      H(3, 8)=4.*X
      H(3,11)=H(1,7)*X
      DO 48 J=1,3
      YBJ=YBAR(J)
      WJ=WBAR(J)
      Y=HALFB*(YBJ+1)
      WIWJ=WI*WJ
C     EVALUATE P(X,Y) AT XI,YJ
C     FIRST EVALUATE H AT XI,YJ
      H(1, 8)=2.*Y
      H(1,11)=H(1,7)*Y
      H(1,12)=H(1,8)*Y
      H(1,14)=H(1,11)*Y
      H(1,15)=H(1,12)*Y
      H(1,16)=H(1,14)*Y
      H(2,10)=6.*Y
```

```
      H(2,13)=H(1,11)
      H(2,15)=H(2,13)*X
      H(2,16)=H(2,15)*X
      H(3, 9)=4.*Y
      H(3,12)=H(2,9)*H(3,9)
      H(3,13)=H(2,10)*Y
      H(3,14)=H(1,8)*H(3,11)
      H(3,15)=H(2,9)*H(3,13)
      H(3,16)=H(1,14)*3.*X
C     COMPUTE PRODUCT OF (H TRANSPOSE)X(D)X(H) X WI X WJ
      DO 40 II=1,16
      DO 40 JJ=1,16
      HTDH(II,JJ)=0.
      DO 30 LL=1,3
      DO 30 KK=1,3
      HTDH(II,JJ)=HTDH(II,JJ)+H(LL,II)*D(LL,KK)*H(KK,JJ)
   30 CONTINUE
      HTDH(II,JJ)=WI*WJ*HTDH(II,JJ)
   40 CONTINUE
C     ADD HTDH INTO TOTAL P MATRIX
      DO 45 II=1,16
      DO 45 JJ=1,16
   45 P(II,JJ)=P(II,JJ)+HTDH(II,JJ)
   48 CONTINUE
C     COMPUTE (G TRANSPOSE)X(INTGRAL P)X(G)
   49 DO 60 II=1,16
      DO 60 JJ=1,16
      DO 50 LL=1,16
      DO 50 KK=1,16
      STIFFM(II,JJ)=STIFFM(II,JJ)+G(LL,II)*P(LL,KK)*G(KK,JJ)
   50 CONTINUE
C     STIFFNESS MATRIX FOR AN ELEMENT NOT ON WELDED EDGE
   52 STIFFM(II,JJ)=ABBET*STIFFM(II,JJ)
   60 CONTINUE
C     STIFFNESS MATRIX IS COMPLETE AND ALL Q'S ARE ZERO
      RETURN
      END
```

```
      SUBROUTINE STIFX(G,STIFFX,QX)
C     ADDITIONAL TERMS FOR STIFFNESS MATRIX FOR AN ELEMENT WITH WELDED
C     EDGE PARALLEL TO X AXIS
      DIMENSION G(16,16),QX(16),STIFFX(16,16)
      COMMON /IO/ KREAD,KWRITE,KPUNCH
      COMMON /GAUSS/ WBAR(3),XBAR(3),YBAR(3)
      COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETAZ,ABBET,HALFA,HALFB,ASQ,ACU
     13,BSQ,BCU3,CXB2,CYA2,BTHCX,ATHCY
      COMMON /EXTRA/ P(16,16),WF(16),FX(16),EKX(16,16)
      CALL ERASE (P,256,WF,16,QX,16,EKX,256)
      Y=B
      FX(1)=0.
      FX(2)=0.
      FX(3)=1.
      FX(5)=0.
      FX(6)=2.*Y
      FX(7)=0.
      FX(10)=3.*Y*Y
      DO 130 JJ=1,3
      WJ=WBAR(JJ)
      CONST=ATHCY*WJ
      X=HALFA*(XBAR(JJ)+1)
      FX(4)=X
      FX(8)=X*X
      FX(9)=FX(5)*X
      FX(11)=X*X*X
      FX(12)=FX(9)*X
      FX(13)=FX(10)*X
      FX(14)=FX(12)*X
      FX(15)=FX(13)*X
      FX(16)=FX(15)*X
      DO 120 KK=1,16
      WF(KK)=WF(KK)+CONST*FX(KK)
      DO 120 LL=1,16
      P(LL,KK)=P(LL,KK)+WJ*FX(LL)*FX(KK)
  120 CONTINUE
```

```
130 CONTINUE
    DO 150 II=1,16
    DO 150 JJ=1,16
    DX(II)=DX(II)+G(JJ,II)*WF(JJ)
    DO 140 KK=1,16
    DO 140 LL=1,16
    EKX(II,JJ)=EKX(II,JJ)+G(LL,II)*P(LL,KK)*G(KK,JJ)
140 CONTINUE
    STIFFX(II,JJ)=CYA2*EKX(II,JJ)
150 CONTINUE
    RETURN
    END
```

```
      SUBROUTINE STIFY(G,STIFFY,QY)
C     ADDITIONAL TERMS FOR STIFFNESS MATRIX FOR AN ELEMENT WITH WELDED
C     EDGE PARALLEL TO Y AXIS
      DIMENSION G(16,16),QY(16),STIFFY(16,16)
      COMMON /IO/ KREAD,KWRITE,KPUNCH
      COMMON /GAUSS/ WBAR(3),XBAR(3),YBAR(3)
      COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETAZ,ABBET,HALFA,HALFB,ASQ,ACU
     1B,BSQ,BCUB,CXB2,CYA2,BTHCX,ATHCY
      COMMON /EXTRA/ P(16,16),WF(16),FY(16),EKY(16,16)
      CALL ERASE (P,256,WF,16,QY,16,EKY,256)
      X=A
      FY(1)=0.
      FY(2)=1.
      FY(3)=0.
      FY(5)=2.*X
      FY(6)=0.
      FY(7)=3.*X*X
      FY(10)=0.
      DO 80 JJ=1,3
      WJ=WBAR(JJ)
      CONST=BTHCX*WJ
      Y=HALFB*(YBAR(JJ)+1)
      FY(4)=Y
      FY(8)=FY(5)*Y
      FY(9)=Y*Y
      FY(11)=FY(7)*Y
      FY(12)=FY(8)*Y
      FY(13)=FY(9)*Y
      FY(14)=FY(11)*Y
      FY(15)=FY(12)*Y
      FY(16)=FY(14)*Y
      DO 70 KK=1,16
      WF(KK)=WF(KK)+CONST*FY(KK)
      DO 70 LL=1,16
      P(LL,KK)=P(LL,KK)+FY(LL)*FY(KK)*WJ
   70 CONTINUE
```

```
80    CONTINUE
      DO 100 II=1,16
      DO 100 JJ=1,16
      QY(II)=QY(II)+G(JJ,II)*WF(JJ)
      DO 90 KK=1,16
      DO 90 LL=1,16
      EKY(II,JJ)=EKY(II,JJ)+G(LL,II)*P(LL,KK)*G(KK,JJ)
90    CONTINUE
      STIFFY(II,JJ)=CXB2*EKY(II,JJ)
100   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ASEMBL(LNUM,NDE,ELK,ELQ,REAL,INTGR)                    ASEM0001
C***********************************************************************ASEM0002
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2            ASEM0003
C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY                        ASEM0004
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY                                 ASEM0005
C***********************************************************************ASEM0006
      DIMENSION REAL(2),INTGR(2)                                        ASEM0007
      DIMENSION ELK(NDE,NDE), ELQ(NDE)                                  ASEM0008
C******************************                                         ASEM0009
C CHANGE THIS DIMENSION STATEMENT IF YOU HAVE RECEIVED AN ERROR MESSAGE ASEM0010
C INDICATING LENGTH OF MNUM VECTOR HAS BEEN EXCEEDED.  ALSO CHANGE      ASEM0011
C INSTRUCTION INDICATED BELOW                                           ASEM0012
      DIMENSION MNUM(100)                                               ASEM0013
C******************************                                         ASEM0014
      COMMON /IO/ KR, KW, KP, KT1, KT2, KT3                             ASEM0015
      COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK                      ASEM0016
C                                                                       ASEM0017
      COMMON /DYNAM/ IKOUM1,IQM1,MNUM,INDEX,I,J,LROW,LCOL,MROW,MCOL,    ASEM0018
     2 KADR,LENGTH                                                      ASEM0019
C                                                                       ASEM0020
C PRINT CONTROL                                                         ASEM0021
  901 FORMAT(1H0,I10,27H DOF/ELEMENT EXCEEDS LENGTH,I4,26H SPECIFIED FORASEM0022
     1 MNUM VECTOR,/,1X,44HERROR OCCURED DURING ASSEMBLY OF ELEMENT NO.,ASEM0023
     2 I10,/,1X,93HUSER MUST CORRECT CALL ARGUMENTS IN MAIN OR CHANGE LEASEM0024
     3NGTH OF MNUM VECTOR IN SUBROUTINE ASEMBL,/,1X,20HEXECUTION TERMINAASEM0025
     4TED)                                                              ASEM0026
      IKOUM1 = IKOUNT-1                                                 ASEM0027
      IQM1 = IQ-1                                                       ASEM0028
C******************************                                         ASEM0029
C VALUE OF LENGTH MUST = DIMENSION OF MNUM                              ASEM0030
      LENGTH = 100                                                      ASEM0031
C******************************                                         ASEM0032
      IF (NDE .LE. LENGTH) GO TO 1                                      ASEM0033
      WRITE (KW,901) NDE, LENGTH, LNUM                                  ASEM0034
      STOP                                                              ASEM0035
C GET MASTER ROW/COL NOS FOR ELEMENT AND STORE IN MNUM                  ASEM0036
```

```
    1 INDEX = IMASTR+LNUM-1                                             ASEM0037
      DO 2 I = 1,NDE                                                    ASEM0038
      J = INTGR(INDEX)+I-1                                              ASEM0039
    2 MNUM(I) = INTGR(J)                                                ASEM0040
C LOOP OVER ROWS OF ELQ AND OVER LOWER TRIANGLE OF ELEMENT K MATRIX      ASEM0041
      DO 4 LROW = 1,NDE                                                 ASEM0042
      INDEX = IQM1+MNUM(LROW)                                           ASEM0043
C ASSEMBLE ELEMENT EQUIVALENT NODAL FORCE INTO Q VECTOR                 ASEM0044
      REAL(INDEX) = REAL(INDEX)+ELQ(LROW)                               ASEM0045
      DO 4 LCOL = 1,LROW                                                ASEM0046
      MROW = MNUM(LROW)                                                 ASEM0047
      MCOL = MNUM(LCOL)                                                 ASEM0048
      IF (MROW .GE. MCOL) GO TO 3                                       ASEM0049
      MROW = MNUM(LCOL)                                                 ASEM0050
      MCOL = MNUM(LROW)                                                 ASEM0051
C CALCULATE ABSOLUTE ADDRESS OF K(MROW,MCOL)                            ASEM0052
    3 INDEX = IKOUM1+MROW                                               ASEM0053
      KADR = INTGR(INDEX)+MCOL                                          ASEM0054
C ASSEMBLE STIFFNESS COEFFICIENT                                        ASEM0055
    4 REAL(KADR) = REAL(KADR)+ELK(LROW,LCOL)                            ASEM0056
      RETURN                                                            ASEM0057
      END                                                               ASEM0058
```

```
      SUBROUTINE BCON(REAL,INTGR)                                       BCON0001
C***********************************************************************BCON0002
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2            BCON0003
C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY                        BCON0004
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY                                 BCON0005
C***********************************************************************BCON0006
      DIMENSION REAL(2),INTGR(2)                                        BCON0007
      COMMON /IO/ KR, KW, KP, KT1, KT2, KT3                             BCON0008
      COMMON /SIZE/ NET, NDT                                            BCON0009
      COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK                      BCON0010
      COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK                        BCON0011
C                                                                       BCON0012
      COMMON /DYNAM/ LAST,IFLAG,I,J,MROW,M,NEXT,NROW,IROW,ICOL,KADR,N,  BCON0013
     2 INIT,MCOL,ILNZM1,IKOUM1,IQM1                                     BCON0014
C                                                                       BCON0015
C PRINT CONTROL                                                         BCON0016
  901 FORMAT(79H0DISPLACEMENT CONSTRAINTS HAVE BEEN APPLIED TO THE FOLLOBCON0017
     1WING DEGREES OF FREEDOM,/,5X,7HDOF NO.,2X,12HDISPLACEMENT)        BCON0018
  902 FORMAT (2X,I10,2X,E10.3)                                          BCON0019
  903 FORMAT(1X,72H***********************ABOVE DOF NUMBER APPEARS TWICEBCON0020
     1 IN CONSTRAINT LIST,/,1X,85HEXECUTION TERMINATED IN SUBROUTINE BCOBCON0021
     2N DUE TO POSSIBLITY OF BOUNDARY CONDITION ERROR)                  BCON0022
  904 FORMAT(62H0YOUR STRUCTURE IS FLYING FREE. PLEASE CONSTRAIN IT NEXTBCON0023
     1 TIME.,/,1X,28HEXECUTION TERMINATED IN BCON)                      BCON0024
      IKOUM1 = IKOUNT-1                                                 BCON0025
      ILNZM1 = ILNZ-1                                                   BCON0026
      IQM1 = IQ-1                                                       BCON0027
C PRINT ENTRY MESSAGE                                                   BCON0028
      WRITE (KW,901)                                                    BCON0029
C ORDER THE CONSTRAINT ROW NUMBERS IN ASCENDING SEQUENCE                BCON0030
      LAST = LCON-1                                                     BCON0031
       IF (LCON .GT. 0) GO TO 1                                         BCON0032
       WRITE (KW,904)                                                   BCON0033
        STOP                                                            BCON0034
    1 IFLAG = 0                                                         BCON0035
      DO 2 I = ICON,LAST                                                BCON0036
```

```
      IF (INTGR(I+1) .GE. INTGR(I)) GO TO 2                             BCON0037
      J = INTGR(I)                                                      BCON0038
      INTGR(I) = INTGR(I+1)                                             BCON0039
      INTGR(I+1) = J                                                    BCON0040
      IFLAG = 1                                                         BCON0041
    2 CONTINUE                                                          BCON0042
      IF (IFLAG .EQ. 1) GO TO 1                                         BCON0043
C CHECK TO SEE IF ANY ROW NUMBERS HAVE BEEN ENTERED IN CONSTRAINT       BCON0044
C VECTOR - ABORT THE RUN IF NONE HAVE BEEN                              BCON0045
      J = 0                                                             BCON0046
      DO 100 I = ICON,LCON                                              BCON0047
  100 J = J+INTGR(I)                                                    BCON0048
      IF (J .GT. 0) GO TO 200                                           BCON0049
      WRITE (KW,904)                                                    BCON0050
      STOP                                                              BCON0051
C OUTPUT CONSTRAINT LIST                                                BCON0052
  200 DO 4 I = ICON,LCON                                                BCON0053
      IF (INTGR(I) .EQ. 0) GO TO 4                                      BCON0054
C CHECK FOR REPEATED DOF AFTER 1ST ONE                                  BCON0055
      IF (I .EQ. ICON) GO TO 3                                          BCON0056
      IF (INTGR(I) .NE. INTGR(I-1)) GO TO 3                             BCON0057
      WRITE (KW,903)                                                    BCON0058
      STOP                                                              BCON0059
    3 J = IQM1+INTGR(I)                                                 BCON0060
      WRITE (KW,902) INTGR(I), REAL(J)                                  BCON0061
    4 CONTINUE                                                          BCON0062
C LOOP OVER CONSTRAINED DOF FOR MODIFICATION OF COMPLETELY ASSEMBLED    BCON0063
C FORCE VECTOR                                                          BCON0064
      DO 71 I = ICON,LCON                                               BCON0065
      IF (INTGR(I) .EQ. 0) GO TO 71                                     BCON0066
C CHECK IF PRESCRIBED DISPLACEMENT = 0 -- IF IT DOES, SKIP FORCE VECTOR BCON0067
      MROW = INTGR(I)                                                   BCON0068
      M = IQM1+MROW                                                     BCON0069
      IF (REAL(M) .EQ. 0.) GO TO 71                                     BCON0070
C DISPL .NE. 0 -- LOOP OVER ALL ROWS TO MODIFY FORCE VECTOR -- SKIP     BCON0071
C CONSTRAINED ROWS (CONTROLLED BY VALUE OF NEXT)                        BCON0072
```

```
      NEXT = ICON                                                        BCON0073
      DO 7 NROW = 1,NDT                                                  BCON0074
      IF (NROW .NE. INTGR(NEXT)) GO TO 5                                 BCON0075
      NEXT = NEXT+1                                                      BCON0076
      GO TO 7                                                            BCON0077
    5 IF (MROW .GT. NROW) GO TO 51                                       BCON0078
C CHECK FOR COUPLING OF ROW NROW WITH COL MROW                            BCON0079
      J = ILNZM1+NROW                                                    BCON0080
      IF (INTGR(J) .GT. MROW) GO TO 7                                    BCON0081
      IROW = NROW                                                        BCON0082
      ICOL = MROW                                                        BCON0083
      GO TO 6                                                            BCON0084
C CHECK FOR COUPLING OF ROW MROW WITH COL NROW                            BCON0085
   51 J = ILNZM1+MROW                                                    BCON0086
      IF (INTGR(J) .GT. NROW) GO TO 7                                    BCON0087
      IROW = MROW                                                        BCON0088
      ICOL = NROW                                                        BCON0089
C SUBTRACT K*(PRESCR DISPL) FROM FORCE VECTOR                             BCON0090
    6 KADR = IKOUM1+IROW                                                 BCON0091
      KADR = INTGR(KADR)+ICOL                                            BCON0092
      N = IQM1+NROW                                                      BCON0093
      REAL(N) = REAL(N)-REAL(KADR)*REAL(M)                               BCON0094
    7 CONTINUE                                                           BCON0095
   71 CONTINUE                                                           BCON0096
C LOOP OVER CONSTRAINED ROWS TO DECOUPLE THEM FROM REST OF K MATRIX       BCON0097
      DO 11 I = ICON,LCON                                                BCON0098
      IF (INTGR(I) .EQ. 0) GO TO 11                                      BCON0099
      MROW = INTGR(I)                                                    BCON0100
      INIT = ILNZM1+MROW                                                 BCON0101
      INIT = INTGR(INIT)                                                 BCON0102
C SET ROW = 0                                                             BCON0103
      M = IKOUM1+MROW                                                    BCON0104
      M = INTGR(M)                                                       BCON0105
      DO 8 MCOL = INIT,MROW                                              BCON0106
      KADR = M+MCOL                                                      BCON0107
    8 REAL(KADR) = 0.                                                    BCON0108
```

```
C SET COLUMN = 0 IN ROWS WHOSE LNZE COL NO IS .LE. MROW -- SKIP THIS     BCON0109
C SECTION IF MROW IS THE LAST ROW                                        BCON0110
      IF (MROW .EQ. NDT) GO TO 10                                        BCON0111
      INIT = MROW+1                                                      BCON0112
      DO 9 NROW = INIT,NDT                                               BCON0113
      N = ILNZM1+NROW                                                    BCON0114
      IF (INTGR(N) .GT. MROW) GO TO 9                                    BCON0115
      KADR = IKOUM1+NROW                                                 BCON0116
      KADR = INTGR(KADR)+MROW                                            BCON0117
      REAL(KADR) = 0.                                                    BCON0118
    9 CONTINUE                                                           BCON0119
C SET DIAGONAL ENTRY = 1                                                 BCON0120
   10 KADR = M+MROW                                                      BCON0121
      REAL(KADR) = 1.                                                    BCON0122
   11 CONTINUE                                                           BCON0123
      RETURN                                                             BCON0124
      END                                                                BCON0125
```

```
      SUBROUTINE FACTSD(REAL,INTGR)
C*****************************************************************************
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C*****************************************************************************
C THIS IS ALSO SUBROUTINE FACTPD - SEE ENTRY POINT BELOW
      DIMENSION REAL(2),INTGR(2)
      COMMON /IO/ KR, KW, KP, KT1, KT2, KT3
      COMMON /SIZE/ NET, NDT
      COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK
      COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK
C
      COMMON /DYNAM/ NPD,I,NEXT,JROW,MPD,TEST,MROW,M,MM,MCOL,SUM,NN,
     2 INIT,N,LAST,J,KADRM,KADRN,JJ,KADR,TESTR,IROW,IERR,IKOUM1,ILNZM1
C
C PRINT CONTROL
  901 FORMAT (1H0,47X,25HTRIPLE FACTOR ENTRY POINT,/,53H K MATRIX NOT PO
     2SITIVE-DEFINITE IN THE FOLLOWING ROWS )
  902 FORMAT (40X,I12)
  903 FORMAT (48X,4HNONE )
  904 FORMAT (58H USER SPECIFIED SEMI-DEFINITE MATRIX.  EXECUTION CONTIN
     2UES )
  905 FORMAT (40H USER SPECIFIED POSITIVE-DEFINITE MATRIX,/,21H EXECUTIO
     2N TERMINATES )
  906 FORMAT (25H0K MATRIX SINGULAR IN ROW,I12,/,21H EXECUTION TERMINATE
     2S )
  907 FORMAT (61H0LARGEST ROUNDING ERROR IN DIAGONAL FACTORING OCCURRED
     2IN ROW,I12,/,43H NUMBER OF LOWEST SIGNIFICANT FIGURES LOST=,I3)
  908 FORMAT (42H ROUNDING ERROR EXCEEDS ACCEPTABLE MAXIMUM,/,21H EXECUT
     2ION TERMINATES )
C SET ENTRY FLAG
      NPD = 0
      GO TO 1
      ENTRY FACTPD(REAL,INTGR)
      NPD = 1
```

```
C PRINT ENTRY MESSAGE                                                    FACT0037
    1 WRITE (KW,901)                                                      FACT0038
      IKOUM1 = IKOUNT-1                                                   FACT0039
      ILNZM1 = ILNZ-1                                                     FACT0040
C INITIALIZE AT 1ST NONZERO ENTRY IN CONSTRAINT VECTOR                    FACT0041
      DO 2 I = ICON,LCON                                                  FACT0042
      IF (INTGR(I) .EQ. 0) GO TO 2                                        FACT0043
      GO TO 3                                                             FACT0044
    2 CONTINUE                                                            FACT0045
    3 NEXT = I                                                            FACT0046
      IF (INTGR(NEXT) .EQ. 1) NEXT = NEXT+1                               FACT0047
C INITIALIZE ERROR PARAMETERS                                             FACT0048
      JROW = 1                                                            FACT0049
      MPD = 0                                                             FACT0050
      TEST = 1.                                                           FACT0051
C DO FIRST ROW AS SPECIAL CASE                                            FACT0052
      IF (REAL(IK) .EQ. 0.) GO TO 14                                      FACT0053
      IF (REAL(IK) .GT. 0.) GO TO 4                                       FACT0054
C NPD MESSAGE AND FLAG                                                    FACT0055
      WRITE (KW,902) JROW                                                 FACT0056
      MPD = 1                                                             FACT0057
C LOOP OVER REMAINING ROWS                                                FACT0058
    4 DO 13 MROW = 2,NDT                                                  FACT0059
C CHECK FOR CONSTRAINED ROW - SKIP IF FOUND AND RESET FOR THE NEXT ONE    FACT0060
      IF (MROW .NE. INTGR(NEXT)) GO TO 5                                  FACT0061
      NEXT = NEXT+1                                                       FACT0062
      GO TO 13                                                            FACT0063
C FREE ROW - FACTOR FROM LNZ COL NO TO ROW NO                             FACT0064
    5 M = ILNZM1+MROW                                                     FACT0065
      M = INTGR(M)                                                        FACT0066
      MM = IKOUM1+MROW                                                    FACT0067
      MM = INTGR(MM)                                                      FACT0068
      DO 12 MCOL = M,MROW                                                 FACT0069
      SUM = 0.                                                            FACT0070
      NN = IKOUM1+MCOL                                                    FACT0071
      NN = INTGR(NN)                                                      FACT0072
```

```
C LNZE IS A SPECIAL CASE - NO SUM REQUIRED                               FACT0073
      IF (MCOL .EQ. M) GO TO 7                                           FACT0074
C START SUM FROM GREATEST OF MROW OR ROW 'MCOL' LNZ COL NOS               FACT0075
      INIT = M                                                           FACT0076
      N = ILNZM1+MCOL                                                    FACT0077
      IF (INTGR(N) .GT. M) INIT = INTGR(N)                               FACT0078
C NO SUM IF ROW 'MCOL' HAS LEADING ZEROS UP TO THE DIAGONAL              FACT0079
      IF (INIT .EQ. MCOL) GO TO 7                                        FACT0080
C ACCUMULATE THE SUM                                                     FACT0081
      LAST = MCOL-1                                                      FACT0082
      DO 6 J = INIT,LAST                                                 FACT0083
      KADRM = MM+J                                                       FACT0084
      KADRN = NN+J                                                       FACT0085
      JJ = IKOUM1+J                                                      FACT0086
      JJ = INTGR(JJ)+J                                                   FACT0087
    6 SUM = SUM+REAL(JJ)*REAL(KADRM)*REAL(KADRN)                         FACT0088
C BRANCH TO SPECIAL ALGORITHM FOR DIAGONAL ENTRIES                       FACT0089
    7 IF (MCOL .EQ. MROW) GO TO 8                                        FACT0090
C FOR OFF-DIAGONAL ENTRIES:                                              FACT0091
      KADR = MM+MCOL                                                     FACT0092
      NN = NN+MCOL                                                       FACT0093
      REAL(KADR) = (REAL(KADR)-SUM)/REAL(NN)                             FACT0094
      GO TO 12                                                           FACT0095
C DIAGONAL ENTRY - TEST FOR SINGULARITY AND SEMI-DEFINITENESS            FACT0096
    8 MM = MM+MROW                                                       FACT0097
      IF (REAL(MM)-SUM .NE. 0.) GO TO 9                                  FACT0098
      JROW = MROW                                                        FACT0099
      GO TO 14                                                           FACT0100
    9 IF (REAL(MM)-SUM .GT. 0.) GO TO 10                                 FACT0101
      WRITE (KW,902) MROW                                                FACT0102
      MPD = 1                                                            FACT0103
C CALCULATE ROUNDING ERROR                                               FACT0104
   10 TESTR = ABS((REAL(MM)-SUM)/REAL(MM))                               FACT0105
      IF (TESTR .GE. TEST) GO TO 11                                      FACT0106
      TEST = TESTR                                                       FACT0107
      IROW = MROW                                                        FACT0108
```

```
C EVALUATE DIAGONAL ENTRY                                               FACT0109
   11 REAL(MM) = REAL(MM)-SUM                                           FACT0110
   12 CONTINUE                                                          FACT0111
   13 CONTINUE                                                          FACT0112
C SEMI-DEFINITENESS CHECKS AND ROUNDING ERROR OUTPUT                     FACT0113
      IF (MPD .EQ. 0) WRITE (KW,903)                                    FACT0114
      IF (MPD .EQ. 1 .AND. NPD .EQ. 1) WRITE (KW,905)                   FACT0115
      IF (MPD .EQ. 1 .AND. NPD .EQ. 0) WRITE (KW,904)                   FACT0116
      IERR = -1.00001*ALOG10(TEST)                                      FACT0117
      WRITE (KW,907) IROW,IERR                                          FACT0118
      IF (IERR .GT. 5) WRITE (KW,908)                                   FACT0119
      IF (MPD .EQ. 1 .AND. NPD .EQ. 1) STOP                             FACT0120
      IF (IERR .GT. 5) STOP                                             FACT0121
      RETURN                                                            FACT0122
C SINGULAR MATRIX                                                       FACT0123
   14 WRITE (KW,906) JROW                                               FACT0124
      STOP                                                              FACT0125
      END                                                               FACT0126
```

```
      SUBROUTINE ORK(LENGTH,REAL,INTGR)
C***********************************************************************
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C***********************************************************************
C     THIS SUBROUTINE CREATES
C              1) THE LNZ VECTOR WHICH HOLDS THE COLUMN NUMBER
C                 OF THE LEADING NON-ZERO ENTRY IN EACH ROW OF
C                 THE ASSEMBLED "K" MATRIX
C              2) THE ADDRESS COUNT VECTOR WHICH HOLDS THE ABSOLUTE
C                 ADDRESS OF THE DIAGONAL ENTRY FOR EACH ROW OF THE
C                 ASSEMBLED K MATRIX, MINUS THE ROW NUMBER, AND
C                 CHECKS THAT K FITS IN THE /DATA/ VECTOR
      DIMENSION REAL(2),INTGR(2)
      COMMON /IO/ KR, KW, KP, KT1, KT2, KT3
      COMMON /SIZE/ NET, NDT
      COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK
      COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK
C
      COMMON /DYNAM/ ILNZM1,IMSTM1,IKOUM1,NETM1,IMSTP1,IROW,MSUB,LNUM,
     2 MADDR,NDE,ISMALL,JDOF,INDEX,NENTRY,DENS,I,J
C
C PRINT CONTROL
  100 FORMAT(49H0THE LENGTH OF THE "DATA" VECTOR FOR THIS CASE IS,I10,14
     1H WHICH EXCEEDS,I10,49H =THE MAXIMUM ALLOWED IN THE DIMENSION STAT
     2EMENT./39H EXECUTION TERMINATED IN SUBROUTINE ORK)
 200  FORMAT(5X,3HROW,2X,13HLNZE COL. NO.,2X,18HABS. ADR. OF DIAG.)
 300  FORMAT(I9,I10,I12)
  400 FORMAT(10H0THERE ARE,I10,68H NON-ZERO ENTRIES IN "K". IF "K" WERE
     1FULLY POPULATED THERE WOULD BE,I10, 9H ENTRIES.,/,20X,15HTHE DENSI
     2TY IS ,E15.6)
      ILNZM1=ILNZ-1
      IMSTM1=IMASTR-1
      IKOUM1=IKOUNT-1
      NETM1=NET-1
```

```
      IMSTP1=IMASTR+1
C SET EACH LNZ COLUMN NO = ROW NO (DIAGONAL MATRIX)
      DO 30 IROW=1,NDT
      MSUB=ILNZM1+IROW
 30   INTGR(MSUB)=IROW
C EXAMINE MASTER ASSEMBLY LIST, ONE ELEMENT AT A TIME, TO CREATE
C THE LNZ VECTOR
      DO 20 LNUM=1,NET
      MADDR = IMSTM1+LNUM
      MADDR = INTGR(MADDR)-1
C CALCULATE NO. DOF IN THE ELEMENT BY DIFFERENCING POINTERS, OR ...
      I = IMASTR+LNUM
      IF(LNUM.EQ.NET) GO TO 3
      NDE = INTGR(I)-INTGR(I-1)
      GO TO 4
C ... BEGIN BY ASSUMING THE LIST IS FILLED, FOR LAST ELEMENT
 3    NDE = LMASTR-INTGR(I-1)+1
C INITIALIZE SMALLEST DOF NO. AT LARGEST POSSIBLE VALUE
 4    ISMALL=NDT
C     FIND SMALLEST MASTR NUMBER FOR THIS ELEMENT
      DO 5 JDOF=1,NDE
      INDEX=MADDR+JDOF
      IF(INTGR(INDEX).GT.ISMALL) GO TO 5
C DISCONTINUE SEARCH IF A ZERO IS FOUND, INDICATING EXCESS STORAGE
C AND PREMATURE END OF LIST FOR LAST ELEMENT
      IF(INTGR(INDEX).EQ.0) GO TO 6
      ISMALL=INTGR(INDEX)
 5    CONTINUE
C     FIND COLUMN NUMBER OF LEADING NON-ZERO ENTRY IN ROW
 6    DO 10 JDOF=1,NDE
      INDEX=MADDR+JDOF
      INDEX=ILNZM1+INTGR(INDEX)
C CHANGE LNZ COL NO ONLY IF NEW ONE IS LESS THAN OLD ONE
      IF(INTGR(INDEX).LT.ISMALL) GO TO 8
      INTGR(INDEX)=ISMALL
      GO TO 10
```

```
C DISCONTINUE OPERATION IF EXCESS STORAGE IS DISCOVERED
 8    IF(INTGR(INDEX).EQ.0) GO TO 20
 10   CONTINUE
 20   CONTINUE
C CREATE ADDRESS COUNT VECTOR
      INTGR(IKOUNT) = IK
      INDEX=IKOUNT
      DO 40 IROW=2,NDT
      I = ILNZM1+IROW
      INTGR(INDEX+1) = INTGR(INDEX)+IROW+1-INTGR(I)
 40   INDEX=INDEX+1
C ADDRESS COUNT VECTOR NOW CONTAINS ABSOLUTE ADDRESS ONLY FOR THE
C DIAGONAL ENTRIES, AND THUS INTGR(LKOUNT) = LK EXACTLY
      IF (INTGR(LKOUNT) .LE. LENGTH) GO TO 50
      WRITE (KW,100) INTGR(LKOUNT), LENGTH
      STOP
 50   LK = INTGR(LKOUNT)
      WRITE(KW,200)
      DO 60 IROW=1,NDT
      I = ILNZM1+IROW
      J = IKOUM1+IROW
      WRITE (KW,300) IROW, INTGR(I), INTGR(J)
C     REPLACE THE ABS. ADDRESS OF DIAG. BY (ABS. ADDRESS - ROW NO.)
      INTGR(J) = INTGR(J)-IROW
 60   CONTINUE
      NENTRY = INTGR(LKOUNT)+NDT-IK+1
      INDEX = (NDT*(NDT+1))/2
      DENS = FLOAT(NENTRY)/FLOAT(INDEX)
      WRITE (KW,400) NENTRY, INDEX, DENS
C ZERO THE FORCE/DISPLACEMENT VECTOR AND THE K MATRIX BLOCK
      DO 70 I=IQ,LK
 70   REAL(I)=0.
      RETURN
      END
```

```
      SUBROUTINE SETUP(LENGTH,NCON,MASTRL,REAL,INTGR)                   SETU0001
C*********************************************************************SETU0002
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2            SETU0003
C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY                        SETU0004
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY                                 SETU0005
C*********************************************************************SETU0006
      DIMENSION REAL(2),INTGR(2)                                        SETU0007
      DIMENSION II(6), LI(6), KD(6)                                     SETU0008
      COMMON /IO/ KR, KW, KP, KT1, KT2, KT3                             SETU0009
      COMMON /SIZE/ NET, NDT                                            SETU0010
      COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK                      SETU0011
      COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK                        SETU0012
C                                                                       SETU0013
      COMMON /DYNAM/ I, TR, DENS                                        SETU0014
      EQUIVALENCE  (ICON, II(1)), (LCON, LI(1)),  (KR, KD(1))           SETU0015
C                                                                       SETU0016
C PRINT CONTROL                                                         SETU0017
  901 FORMAT (1H0,53X,11HSETUP ENTRY,/,42X,35HUSER SPECIFICATIONS TO FEASETU0018
     1BL SYSTEM,/,1H0,51X,16HI/O DEVICE CODES,/,27X,6HREADER,5X,7HPRINTESETU0019
     2R,2X,10HCARD PUNCH,7X,5HTAPE1,7X,5HTAPE2,7X,5HTAPE3,/,21X,6(2X,I10SETU0020
     3),/,1H0,53X,12HPROBLEM SIZE,/,42X,25HTOTAL NUMBER OF ELEMENTS=,I10SETU0021
     4,/,42X,25HTOTAL DEGREES OF FREEDOM=,I10,/,1H0,51X,16HENTRY PARAMETSETU0022
     5ERS,/,39X,32HASSUMED LENGTH OF /DATA/ VECTOR=,I10,/,32X,45HNUMBER SETU0023
     6OF DISPLACEMENT CONSTRAINTS REQUESTED=,I10,/,33X,44HNUMBER OF WORDSETU0024
     7S REQUESTED FOR ASSEMBLY LIST=,I10)                               SETU0025
  902 FORMAT (1H0,43X,31H/DATA/ VECTOR ADDRESS INDEX MAP,/,22X,11HCONSTRSETU0026
     1AINTS,2X,10HDG ABS ADR,1X,11HLNZE COL NO,1X,11HASMBLY LIST,2X,10HQSETU0027
     2/U VECTOR,4X,8HK MATRIX,/,16X,5HBEGIN,6(2X,I10),/,18X,3HEND,5(2X,ISETU0028
     310),11X,1H+,/,1H0,36H+ LK IS CALCULATED BY SUBROUTINE CRK)        SETU0029
  903 FORMAT (40HOSTORAGE EXCEEDS LENGTH OF /DATA/ VECTOR,/,1X,34HLENGTHSETU0030
     1 SUGGESTED FOR THIS PROBLEM=,I12,6H WORDS,/,1X,40HEXECUTION TERMINSETU0031
     2ATED IN SUBROUTINE SETUP)                                         SETU0032
C *********************************************************************SETU0033
C REMOVE THIS FORMAT AND WRITE STATEMENT INDICATED BELOW IF FEABL       SETU0034
C HEADING IS NOT DESIRED                                                SETU0035
 1001 FORMAT(1H1/3(1H ,92(1HX)/),4(5H XXXX,84X,4HXXXX/),5H XXXX,7X,10(1HSETU0036
```

```
     1F),5X,10(1HE),9X,2HAA,9X,8(1HB),7X,3HLLL,14X,4HXXXX/5H XXXX,7X,10(SETU0037
     21HF),5X,10(1HE),8X,4HAAAA,8X,9(1HB),6X,3HLLL,14X,4HXXXX/5H XXXX,7XSETU0038
     3,3HFFF,12X,3HEEE,14X,6(1HA),7X,3HBBB,3X,4HBBBB,5X,3HLLL,14X,4HXXXXSETU0039
     4/5H XXXX,7X,3HFFF,12X,3HEEE,13X,3HAAA,2X,3HAAA,6X,3HBBB,4X,3HBBB,5SETU0040
     5X,3HLLL,14X,4HXXXX/5H XXXX,7X,3HFFF,12X,3HEEE,12X,3HAAA,4X,3HAAA,5SETU0041
     6X,3HBBB,4X,3HBBB,5X,3HLLL,14X,4HXXXX/5H XXXX,7X,3HFFF,12X,3HEEE,12SETU0042
     7X,3HAAA,4X,3HAAA,5X,3HBBB,3X,4HBBBB,5X,3HLLL,14X,4HXXXX/5H XXXX,7XSETU0043
     8,8(1HF),7X,8(1HE),7X,3HAAA,4X,3HAAA,5X,9(1HB),6X,3HLLL,14X,4HXXXX/SETU0044
     95H XXXX,7X,8(1HF),7X,8(1HE),7X,10(1HA),5X,9(1HB),6X,3HLLL,14X,4HXXSETU0045
     AXX/5H XXXX,7X,3HFFF,12X,3HEEE,12X,10(1HA),5X,3HBBB,3X,4HBBBB,5X,3HSETU0046
     BLLL,14X,4HXXXX/5H XXXX,7X,3HFFF,12X,3HEEE,12X,3HAAA,4X,3HAAA,5X,3HSETU0047
     CBBB,4X,3HBBB,5X,3HLLL,14X,4HXXXX/5H XXXX,7X,3HFFF,12X,3HEEE,12X,3HSETU0048
     DAAA,4X,3HAAA,5X,3HBBB,3X,4HBBBB,5X,3HLLL,14X,4HXXXX/5H XXXX,7X,3HFSETU0049
     EFF,12X,10(1HE),5X,3HAAA,4X,3HAAA,5X,9(1HB),6X,10(1HL),7X,4HXXXX/5HSETU0050
     F XXXX,7X,3HFFF,12X,10(1HE),5X,3HAAA,4X,3HAAA,5X,8(1HB),7X,10(1HL),SETU0051
     G7X,4HXXXX/4(5H XXXX,84X,4HXXXX/),5H XXXX,24X,37HFINITE ELEMENT ANASETU0052
     HLYSIS BASIC LIBRARY,23X,4HXXXX/4(5H XXXX,84X,4HXXXX/),3(1H ,92(1HXSETU0053
     I)/),1H1)                                                          SETU0054
C ********************************************************************SETU0055
C PRINT ENTRY MESSAGE                                                 SETU0056
C ********************************************************************SETU0057
C REMOVE THIS WRITE STATEMENT IF FEABL HEADING IS NOT DESIRED         SETU0058
      WRITE (KW,1001)                                                 SETU0059
C ********************************************************************SETU0060
      WRITE (KW,901) (KD(I), I = 1,6), NET, NDT, LENGTH, NCON, MASTRL SETU0061
C CALCULATE ADDRESS INDEX VALUES                                      SETU0062
      ICON = 1                                                        SETU0063
      LCON = NCON                                                     SETU0064
      IKOUNT = LCON+1                                                 SETU0065
      LKOUNT = LCON+NDT                                               SETU0066
      ILNZ = LKOUNT+1                                                 SETU0067
      LLNZ = LKOUNT+NDT                                               SETU0068
      IMASTR = LLNZ+1                                                 SETU0069
      LMASTR = LLNZ+MASTRL                                            SETU0070
      IQ = LMASTR+1                                                   SETU0071
      LQ = LMASTR+NDT                                                 SETU0072
```

```
      IK = LQ+1                                                         SETU0073
C PRINT INDEX MAP                                                       SETU0074
      WRITE (KW,902) (II(I), I = 1,6), (LI(I), I = 1,5)                 SETU0075
C STORAGE BOUNDS TEST                                                   SETU0076
      IF (LMASTR .LE. LENGTH) GO TO 1                                   SETU0077
C STORAGE EXCEEDED - ESTIMATE REQUIRED LENGTH BASED ON LOWER TRIANGLE   SETU0078
C ESTIMATED POPULATION FACTOR                                           SETU0079
      TR = (NDT*(NDT+1))/2                                              SETU0080
      DENS = 0.5                                                        SETU0081
      IF (NDT .GT. 200) DENS=0.3                                        SETU0082
      IF (NDT .GT. 500) DENS=0.2                                        SETU0083
      IF (NDT .GT. 1500) DENS = 0.15                                    SETU0084
      IF (NDT .GT. 2000) DENS = 0.10                                    SETU0085
      TR = TR*DENS                                                      SETU0086
      LENGTH = LQ+TR                                                    SETU0087
      WRITE (KW,903) LENGTH                                             SETU0088
      STOP                                                              SETU0089
C ENOUGH STORAGE EXISTS TO GO THRU ORK                                  SETU0090
    1 DO 2 I = ICON,LCON                                                SETU0091
    2 INTGR(I) = 0                                                      SETU0092
      RETURN                                                            SETU0093
      END                                                               SETU0094
```

```
      SUBROUTINE SIMULQ(ENERGY,REAL,INTGR)
C*****************************************************************************
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABCRATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C*****************************************************************************
      DIMENSION REAL(2),INTGR(2)
      COMMON /IO/ KR, KW, KP, KT1, KT2, KT3
      COMMON /SIZE/ NET, NDT
      COMMON /BEGIN/ ICON, IKOUNT, ILNZ, IMASTR, IQ, IK
      COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK
C
      COMMON /DYNAM/ IKOUM1,ILNZM1,IQM1,IFLAG,MOST,LEFT,JBEG,NBEG,I,
     2 JEND,NEND,J,N,NEXT,MROW,SUM,LAST,M,MM,KADR,INIT
C
C PRINT CONTROL
  901 FORMAT (1H0,53X,18HSIMULQ ENTRY POINT,/,38H PRESCRIBED FORCE/DISPL
     2ACEMENT VECTOR: )
  902 FORMAT (30H0DISPLACEMENT SOLUTION VECTOR: )
  903 FORMAT (1H0,2X,3HROW,10I12)
  904 FORMAT (6H VALUE,10(2X,E10.3))
  905 FORMAT (32H0STRAIN ENERGY IN THE STRUCTURE=,E10.3)
      IKOUM1 = IKOUNT-1
      ILNZM1 = ILNZ-1
      IQM1 = IQ-1
C PRINT ENTRY MESSAGE, SET PRINT SECTION CONTROL FLAG
      WRITE (KW,901)
      IFLAG = 0
C PRINT SECTION:  DIVIDE Q VECTOR INTO GROUPS OF TEN
C PLUS A POSSIBLE REMAINDER
      MOST = NDT/10
      LEFT = NDT-10*MOST
      JBEG = 1
      NBEG = IQ
C RE-ENTRY POINT FOR SOLUTION OUTPUT
    1 IF (NDT .LT. 10) GO TO 3
```

```
      DO 2 I = 1,MOST
      JBEG = 1+10*(I-1)
      JEND = JBEG+9
      NBEG = IQ+JBEG-1
      NEND = NBEG+9
      WRITE (KW,903) (J, J = JBEG,JEND)
    2 WRITE (KW,904) (REAL(N), N = NBEG,NEND)
C CHECK FOR EXISTENCE OF REMAINDER
      IF (LEFT .EQ. 0) GO TO 4
      JBEG = JEND+1
      NBEG = NEND+1
    3 WRITE (KW,903) (J, J = JBEG,NDT)
      WRITE (KW,904) (REAL(N), N = NBEG,LQ)
C CHECK CONTROL FLAG
    4 IF (IFLAG .EQ. 0) GO TO 5
      WRITE (KW,905) ENERGY
      RETURN
C FORWARD SOLUTION - NO DIVISIONS, SO SKIP FIRST ROW ENTIRELY
C INITIALIZE CONSTRAINT VECTOR AT 1ST NONZERO ENTRY
    5 DO 6 I = ICON,LCON
      IF (INTGR(I) .EQ. 0) GO TO 6
      GO TO 7
    6 CONTINUE
    7 NEXT = I
      IF (INTGR(NEXT) .EQ. 1) NEXT = NEXT+1
C SOLVE (A)(R) = (Q)
      DO 10 MROW = 2,NDT
C CHECK FOR CONSTRAINT TO SKIP ROW
      IF (MROW .NE. INTGR(NEXT)) GO TO 8
C UPDATE NEXT
      NEXT = NEXT+1
      GO TO 10
C INITIALIZE SUM AND LOOP LIMITS FOR SUM
    8 SUM = 0.
      LAST = MROW-1
      M = ILNZ+LAST
```

```
      M = INTGR(M)
      MM = IKOUNT+LAST
      MM = INTGR(MM)
      DO 9 J = M,LAST
      KADR = MM+J
      N = IQM1+J
    9 SUM = SUM+REAL(KADR)*REAL(N)
C SUBTRACT SUM FROM Q
      N = IQM1+MROW
      REAL(N) = REAL(N)-SUM
   10 CONTINUE
C SOLVE (D)(P) = (R) AND CALCULATE ENERGY
      N = IQ-1
      ENERGY = 0.
      DO 11 MROW = 1,NDT
      KADR = IKOUM1+MROW
      KADR = INTGR(KADR)+MROW
      N = N+1
      REAL(N) = REAL(N)/REAL(KADR)
   11 ENERGY = ENERGY+REAL(KADR)*(REAL(N)**2)
      ENERGY = 0.5*ENERGY
C BACK SOLUTION - NO DIVISIONS, SO SKIP LAST ROW ENTIRELY
      NEXT = LCON
      IF (INTGR(NEXT) .EQ. NDT) NEXT = NEXT-1
C LOOP BACKWARDS OVER REMAINING ROWS
      DO 14 I = 2,NDT
      MROW = NDT+1-I
C CHECK FOR CONSTRAINT TO SKIP ROW
      IF (MROW .NE. INTGR(NEXT)) GO TO 12
C UPDATE NEXT
      NEXT = NEXT-1
      IF (NEXT .LT. ICON) NEXT = LCON
      GO TO 14
C INITIALIZE SUM AND LOWER LOOP LIMIT
   12 SUM = 0.
      INIT = MROW+1
```

```
      DO 13 J = INIT,NDT                                                SIMU0109
C CHECK IF LNZ COL NO OF ROW J EXCEEDS MROW - IF SO, SKIP                SIMU0110
      N = ILNZM1+J                                                      SIMU0111
      IF (INTGR(N) .GT. MROW) GO TO 13                                  SIMU0112
      KADR = IKOUM1+J                                                   SIMU0113
      KADR = INTGR(KADR)+MROW                                           SIMU0114
      N = IQM1+J                                                        SIMU0115
      SUM = SUM+REAL(KADR)*REAL(N)                                      SIMU0116
   13 CONTINUE                                                          SIMU0117
C SUBTRACT SUM FROM Q                                                   SIMU0118
      N = IQM1+MROW                                                     SIMU0119
      REAL(N) = REAL(N)-SUM                                             SIMU0120
   14 CONTINUE                                                          SIMU0121
C PRINT SOLUTION HEADING, RESET CONTROL FLAG AND                        SIMU0122
C BRANCH TO OUTPUT SECTION                                              SIMU0123
      WRITE (KW,902)                                                    SIMU0124
      IFLAG = 1                                                         SIMU0125
      GO TO 1                                                           SIMU0126
      END                                                               SIMU0127
```

```
      SUBROUTINE XTRACT(LNUM,NDE,ELQ,REAL,INTGR)                        XTRA0001
C***********************************************************************XTRA0002
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2            XTRA0003
C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY                        XTRA0004
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY                                 XTRA0005
C***********************************************************************XTRA0006
      DIMENSION REAL(2),INTGR(2)                                        XTRA0007
      DIMENSION ELQ(NDE)                                                XTRA0008
      COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK                      XTRA0009
      COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK                        XTRA0010
C                                                                       XTRA0011
      COMMON /DYNAM/ ISTART,I,J                                         XTRA0012
C                                                                       XTRA0013
C FIND STARTING LOCATION OF ELEMENT ASSEMBLY LIST, -1                   XTRA0014
      ISTART = IMASTR+LNUM-1                                            XTRA0015
      ISTART = INTGR(ISTART)-1                                          XTRA0016
C EXTRACT DISPLACEMENTS FROM Q AND STORE IN ELQ                         XTRA0017
      DO 1 I = 1,NDE                                                    XTRA0018
      J = ISTART+I                                                      XTRA0019
      J = IQ+INTGR(J)-1                                                 XTRA0020
    1 ELQ(I) = REAL(J)                                                  XTRA0021
      RETURN                                                            XTRA0022
      END                                                               XTRA0023
```

APPENDIX E

Sample Output

DISTORTION ANALYSIS OF WELDED PANEL STRUCTURE....BY D. SHIN
LX= 0.16000E 02, LY= 0.10000E 02, M=NO. ELEMENTS X DIR= 8
NO. ELEMENTS Y DIR= 5, CX= 0.11300E 05, CY= 0.11300E 05
THETA 0= 0.45000E-01 RADIANS, POISSONS RATIO= 0.30000E 00
THICKNESS= 0.37500E 00, YOUNGS MODULUS= 0.30000E 08

NODE	W	DW/DX	DW/DY	D(DW/DY)/DX
1	-0.91928E-01	0.0	0.0	0.0
2	-0.88355E-01	0.0	0.35772E-02	0.0
3	-0.77587E-01	0.0	0.72081E-02	0.0
4	-0.59449E-01	0.0	0.10954E-01	0.0
5	-0.33680E-01	0.0	0.14841E-01	0.0
6	0.0	0.0	0.18860E-01	0.0
7	-0.91103E-01	0.83277E-03	0.0	0.0
8	-0.87579E-01	0.78365E-03	0.35294E-02	-0.53047E-04
9	-0.76942E-01	0.65235E-03	0.71237E-02	-0.90185E-04
10	-0.58998E-01	0.45744E-03	0.10845E-01	-0.11218E-03
11	-0.33455E-01	0.22819E-03	0.14724E-01	-0.11746E-03
12	0.0	0.0	0.18753E-01	-0.10866E-03
13	-0.88539E-01	0.17606E-02	0.0	0.0
14	-0.85157E-01	0.16631E-02	0.33897E-02	-0.10264E-03
15	-0.74923E-01	0.13905E-02	0.68673E-02	-0.18373E-03
16	-0.57579E-01	0.97915E-03	0.10509E-01	-0.23550E-03
17	-0.32746E-01	0.49051E-03	0.14360E-01	-0.25204E-03
18	0.0	0.0	0.18416E-01	-0.23381E-03
19	-0.83915E-01	0.29142E-02	0.0	0.0
20	-0.80781E-01	0.27593E-02	0.31448E-02	-0.15960E-03
21	-0.71257E-01	0.23174E-02	0.64100E-02	-0.29382E-03
22	-0.54990E-01	0.16415E-02	0.99026E-02	-0.38679E-03
23	-0.31445E-01	0.82650E-03	0.13695E-01	-0.42260E-03
24	0.0	0.0	0.17798E-01	-0.39422E-03
25	-0.76646E-01	0.44285E-02	0.0	0.0
26	-0.73887E-01	0.42048E-02	0.27736E-02	-0.22747E-03
27	-0.65447E-01	0.35564E-02	0.57058E-02	-0.42997E-03
28	-0.50856E-01	0.25432E-02	0.89496E-02	-0.58395E-03
29	-0.29355E-01	0.12916E-02	0.12632E-01	-0.65582E-03
30	0.0	0.0	0.16801E-01	-0.61803E-03
31	-0.65869E-01	0.64462E-02	0.0	0.0
32	-0.63630E-01	0.61465E-02	0.22550E-02	-0.30303E-03
33	-0.56724E-01	0.52594E-02	0.46998E-02	-0.59186E-03
34	-0.44567E-01	0.38252E-02	0.75426E-02	-0.84170E-03
35	-0.26136E-01	0.19748E-02	0.11009E-01	-0.99117E-03

36	0.0	0.0	0.15257E-01	-0.95434E-03
37	-0.50438E-01	0.91026E-02	0.0	0.0
38	-0.48869E-01	0.87341E-02	0.15847E-02	-0.37215E-03
39	-0.43977E-01	0.76144E-02	0.33565E-02	-0.75791E-03
40	-0.35158E-01	0.57086E-02	0.55600E-02	-0.11564E-02
41	-0.21198E-01	0.30509E-02	0.85710E-02	-0.14943E-02
42	0.0	0.0	0.12358E-01	-0.15218E-02
43	-0.28973E-01	0.12489E-01	0.0	0.0
44	-0.28182E-01	0.12086E-01	0.80030E-03	-0.40724E-03
45	-0.25692E-01	0.10828E-01	0.17212E-02	-0.86705E-03
46	-0.21094E-01	0.85499E-02	0.29518E-02	-0.14512E-02
47	-0.13388E-01	0.49515E-02	0.49353E-02	-0.22364E-02
48	0.0	0.0	0.88895E-02	-0.27871E-02
49	0.0	0.16597E-01	0.0	0.0
50	0.0	0.16219E-01	0.0	-0.38120E-03
51	0.0	0.15027E-01	0.0	-0.82971E-03
52	0.0	0.12794E-01	0.0	-0.14703E-02
53	0.0	0.88826E-02	0.0	-0.27754E-02
54	0.0	0.0	0.0	-0.85932E-02

DISTORTION ANALYSIS OF WELDED PANEL STRUCTURE....BY D. SHIN
LX= 0.16000E 02, LY= 0.10000E 02, M=NO. ELEMENTS X DIR= 8
NO. ELEMENTS Y DIR= 5, CX= 0.42000E 05, CY= 0.42000E 05
THETA 0= 0.55000E-01 RADIANS, POISSONS RATIO= 0.30000E 00
THICKNESS= 0.37500E 00, YOUNGS MODULUS= 0.30000E 08

NODE	W	DW/DX	DW/DY	D(DW/DY)/DX
1	-0.20630E 00	0.0	0.0	0.0
2	-0.19811E 00	0.0	0.81999E-02	0.0
3	-0.17348E 00	0.0	0.16446E-01	0.0
4	-0.13228E 00	0.0	0.24772E-01	0.0
5	-0.74396E-01	0.0	0.33102E-01	0.0
6	0.0	0.0	0.41245E-01	0.0
7	-0.20479E 00	0.15373E-02	0.0	0.0
8	-0.19671E 00	0.14253E-02	0.80926E-02	-0.12003E-03
9	-0.17237E 00	0.11358E-02	0.16265E-01	-0.19459E-03
10	-0.13156E 00	0.73612E-03	0.24561E-01	-0.21924E-03
11	-0.74079E-01	0.32457E-03	0.32912E-01	-0.19216E-03
12	0.0	0.0	0.41119E-01	-0.13083E-03
13	-0.19997E 00	0.33650E-02	0.0	0.0
14	-0.19222E 00	0.31367E-02	0.77720E-02	-0.23848E-03
15	-0.16878E 00	0.25173E-02	0.15705E-01	-0.40788E-03
16	-0.12922E 00	0.16449E-02	0.23897E-01	-0.47647E-03
17	-0.73044E-01	0.73174E-03	0.32306E-01	-0.43069E-03
18	0.0	0.0	0.40708E-01	-0.29460E-03
19	-0.19090E 00	0.58546E-02	0.0	0.0
20	-0.18374E 00	0.54795E-02	0.71915E-02	-0.38374E-03
21	-0.16195E 00	0.44364E-02	0.14669E-01	-0.67925E-03
22	-0.12473E 00	0.29342E-02	0.22639E-01	-0.82320E-03
23	-0.71033E-01	0.13205E-02	0.31135E-01	-0.76912E-03
24	0.0	0.0	0.39906E-01	-0.53212E-03
25	-0.17587E 00	0.93857E-02	0.0	0.0
26	-0.16963E 00	0.88291E-02	0.62826E-02	-0.56339E-03
27	-0.15045E 00	0.72449E-02	0.13005E-01	-0.10340E-02
28	-0.11705E 00	0.48832E-02	0.20552E-01	-0.13141E-02
29	-0.67547E-01	0.22379E-02	0.29125E-01	-0.12862E-02
30	0.0	0.0	0.38504E-01	-0.90881E-03
31	-0.15242E 00	0.14326E-01	0.0	0.0
32	-0.14748E 00	0.13576E-01	0.49387E-02	-0.75691E-03
33	-0.13209E 00	0.11375E-01	0.10551E-01	-0.14575E-02
34	-0.10449E 00	0.79106E-02	0.17302E-01	-0.19902E-02
35	-0.61697E-01	0.37420E-02	0.25812E-01	-0.21083E-02

36	0.0	0.0	0.36123E-01	-0.15539E-02
37	-0.11745E 00	0.20934E-01	0.0	0.0
38	-0.11416E 00	0.20047E-01	0.33369E-02	-0.89815E-03
39	-0.10373E 00	0.17337E-01	0.72541E-02	-0.18427E-02
40	-0.84243E-01	0.12707E-01	0.12542E-01	-0.28108E-02
41	-0.51826E-01	0.63954E-02	0.20411E-01	-0.34666E-02
42	0.0	0.0	0.31996E-01	-0.28367E-02
43	-0.67591E-01	0.29178E-01	0.0	0.0
44	-0.66083E-01	0.28315E-01	0.15351E-02	-0.87708E-03
45	-0.61216E-01	0.25567E-01	0.34282E-02	-0.19292E-02
46	-0.51732E-01	0.20381E-01	0.62956E-02	-0.34149E-02
47	-0.34341E-01	0.11729E-01	0.11718E-01	-0.56074E-02
48	0.0	0.0	0.24053E-01	-0.64475E-02
49	0.0	0.38519E-01	0.0	0.0
50	0.0	0.37908E-01	0.0	-0.62151E-03
51	0.0	0.35921E-01	0.0	-0.14222E-02
52	0.0	0.31951E-01	0.0	-0.27916E-02
53	0.0	0.24050E-01	0.0	-0.64398E-02
54	0.0	0.0	0.0	-0.28044E-01

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Thesis
S474 Shin

134736

Finite element analysis of out-of-plane distortion of welded panel structures.